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# Powers of numbers.

Cyclicity of powers.

| last digit | Power = 1 | Power = 2 | Power = 3 | Power = 4 | Power = 5 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 4 | 8 (10-2) | 6 | 2. |
| 3 | 3 | 9 | 7. (10-3) | 1 | 3 |
| 4 | 4 | 6. (10-4) | 4. | 6 | 4 |
| 5 | 5 | 5 | 5. | 5 | 5 |
| 6 | 6 | 6 | 6. | 6. | 6 |
| 7 | 7 | 9 | 3 (10-7) | 1 | 7 |
| 8 | 8 | 4 | 2 (10-8) | 6 | 8 |
| 9 | 9 | 1 (10-9) | 9. | 1 | 9. |

$4^{odd} \to 4$, $4^{even} \to 6$.

$9^{odd} \to 9$, $9^{even} \to 1$

perfect squares.

$2 \to$ 2, 4, 8, 6. (10-2) (10-4)

$8 \to$ 8, 4, 2, 6.

$3 \to$ 3, 9, 7, 1

$7 \to$ 7, 9, 3, 1

Numbers not present in the last digit of power 2

= 2, 3, 7, 8.

$\Rightarrow$ no perfect square can end in 2, 3, 7, 8.

Find the last digit of $2^{29}$

$\text{Rem}\left(\frac{29}{4}\right) = 1$.

$2^{29} \equiv 2^1 = 2$.

Find the last digit of $3^{99} \equiv 3^3 = 7$.

Find the last digit of $\cancel{292}^{947}$

$2^{947} \equiv 2^3 = 8$.

Find the last digit of $32^{59} \times 23^{95}$

$2^{59} \times 3^{95} \equiv 2^3 \times 3^3 \equiv 8 \times 7 \equiv \underline{6}$

## Find the no of digits of a power.

$2^5 = 32$ (2) $2^6 = 64$ (2) $2^7 = 128$ (3) $2^8 = 256$ (3)

$2^9 = 512$ (3) $2^{10} = 1024$. (4) $2^{100} =$

$\boxed{\log 2 = 0.301}$

$\log(2^5) = 5 \log 2 = 5 \times 0.301 = 1.505$. no of digits = integer part +1 = 1+1 = 2.

$\log(a^b) = b \log a$.

$\log(2^6) = 6 \log 2 = 6 \times 0.301 = 1.806$.

$\log(2^7) = 7 \log 2 = 7 \times 0.301 = 2.107$

$\log(2^{10}) = 10 \log 2 = 10 \times 0.301 = 3.01$

$\log(2^7) = 7\log 2 = 7\times 0.301 = 2.107$

$\log(2^{10}) = 10\log 2 = 10\times 0.301 = 3.01$

$2^{100} \to 31$ digits $\qquad \log(2^{100}) = 100\log 2 = 100\times 0.301 = 30.1$

$\boxed{\log 3 = 0.4771}$

find the no of digits of $3^{99} \to 48$ digits

$\log(3^{99}) = 99\log 3 = 99\times 0.4771 = 47.2329$

find the no of digits of $4^{100} \to 61$ digits.

$4^{100} = (2^2)^{100} = 2^{200}$.

$200\log 2 = 200\times 0.301 = 60.2$.

$\log 4 = \log(2^2)$
$= 2\log 2$
$= 2\times 0.301$
$= 0.602$.

$\log(ab) = \log a + \log b$.

$\log\left(\frac{a}{b}\right) = \log a - \log b$.

$\log 6 = \log(2\times 3) = \log 2 + \log 3$.
$= 0.301 + 0.4771$
$= 0.7781$.

$\log 5 = \log\left(\frac{10}{2}\right)$
$= \log 10 - \log 2$.
$= 1 - 0.301$
$= 0.699$.

$\boxed{\log 7 = 0.8541}$

$\log 8 = \log(2^3) = 3\log 2 = 3\times 0.301 = 0.903$.

$\log 9 = \log(3^2) = 2\log 3 = 2\times 0.4771 = 0.9542$.

## Remainders

Find the remainder of $\frac{2^9}{7}$.

$R\left(\frac{2^9}{7}\right) = R\left(\frac{(2^3)^3}{7}\right) = \left(R\left(\frac{2^3}{7}\right)\right)^3 = 1^3 = 1$

$2^9 = 512$.

$R\left(\frac{512}{7}\right) = 1$

$R\left(\frac{2^9}{9}\right) = \left[R\left(\frac{2^3}{9}\right)\right]^3 = (-1)^3$.
$= -1$
$9 - 1 = 8$

$R\left(\frac{512}{9}\right) = 8$.

$8 = 9\times 0 + 8$.
$= 9\times 1 - 1$

$17 = 9\times 1 + 8$.
$= 9\times 2 - 1$

Remainder

$9 + (-1) = 8$

Find the remainder of $3^{92} \div 28$.

$3^{92} = 3^{90}\times 3^2$

$R\left(\frac{27}{28}\right) = -1 \qquad R\left(\frac{3^3}{28}\right) = -1 \qquad 3^{90} = (3^3)^{30}$.

Rem of $3^{92} \div 28$
$= (\text{Rem of } 3^{90} \div 28)\times(\text{Rem of } 3^2 \div 28)$

$R\left(\frac{3^{90}}{28}\right) = \left[R\left(\frac{3^3}{28}\right)\right]^{30} = (-1)^{30} = 1$

$\frac{2^5}{3} = \frac{32}{3} \to 2$

$2^5 = 2^4\times 2^1$

$R\left(\frac{2^4}{3}\right) = 1 \qquad R\left(\frac{2}{3}\right) = 2$