02 January 2024 21:10

Q. The sum of the series: $1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \cdots\cdots$ is:

(a) $\frac{3e}{2}$ (b) $\frac{3e}{4}$ (c) $\frac{3e^2}{2}$ (d) $\frac{3e^2}{4}$

$$\underbrace{(1)}_{t_1} + \underbrace{\left(\frac{1+2}{2!}\right)}_{t_2} + \underbrace{\left(\frac{(1+2+3)}{3!}\right)}_{t_3} + \underbrace{\left(\frac{1+2+3+4}{4!}\right)}_{t_4} + \cdots$$

$$t_n = \frac{(1+2+\cdots+n)}{n!} = \frac{n(n+1)/2}{n!} = \frac{n(n+1)}{2\cdot n!} = \frac{n+1}{2(n-1)!}$$

$$t_n = \frac{1}{2}\left[\frac{n-1+2}{(n-1)!}\right] = \frac{1}{2}\left[\frac{(n-1)}{(n-1)!} + \frac{2}{(n-1)!}\right]$$

$$= \frac{1}{2}\cdot\frac{1}{(n-2)!} + \frac{1}{(n-1)!}$$

$$\sum_{n=1}^{\infty} t_n = \frac{1}{2}\sum_{n=2}^{\infty}\frac{1}{(n-2)!} + \sum_{n=1}^{\infty}\frac{1}{(n-1)!}$$

$$= \frac{1}{2}\Big[\underbrace{1 + \frac{1}{1!} + \frac{1}{2!} + \cdots}_{=e}\Big] + \Big[\underbrace{1 + \frac{1}{1!} + \frac{1}{2!} + \cdots}_{=e}\Big]$$

Note: $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots\cdots$

Put $x = 1 \Rightarrow e = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots\cdots$

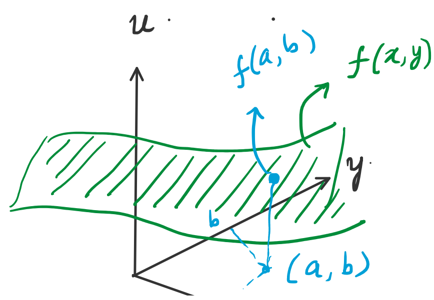
$= \frac{1}{2}\cdot e + e = \frac{3}{2}e$. (a).

Limit & Continuity for Fn of 2 variables

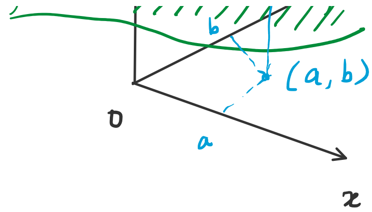
Eg: $u = f(x, y)$

u → dependent variables

x, y → Independent variables.



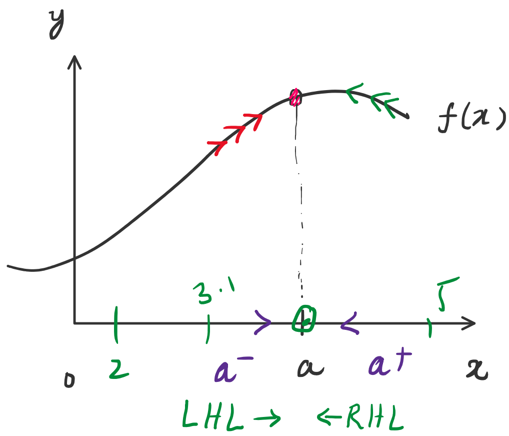
∴ Check whether:



∴ Check whether:

$$\lim_{(x,y)\to(a,b)} u(x,y) \text{ exists or not.}$$

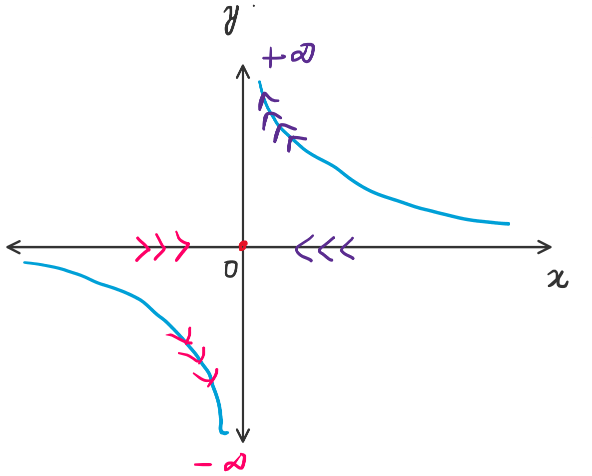
Note: For fn $y = f(x)$, check if fn has limit at $\boxed{x = a}$:-



∴ LHL: $\lim_{x\to a^-} f(x)$

RHL: $\lim_{x\to a^+} f(x)$.

∴ If LHL = RHL ⇒ Limit exists at pt $x = a$.



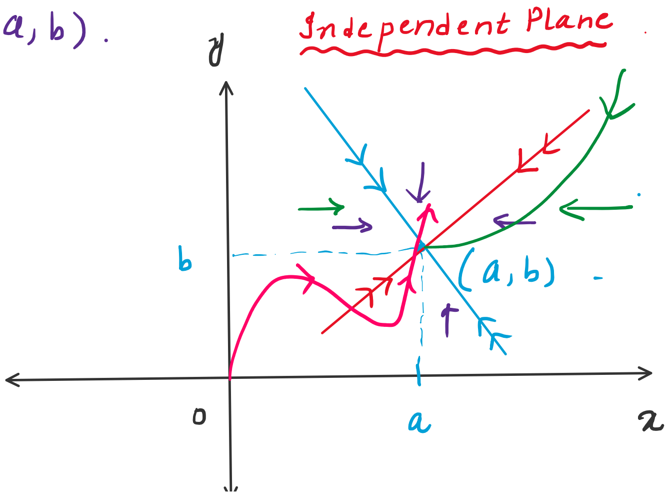
Eg: $y = \dfrac{1}{x}$. Check for at pt $x = 0$.

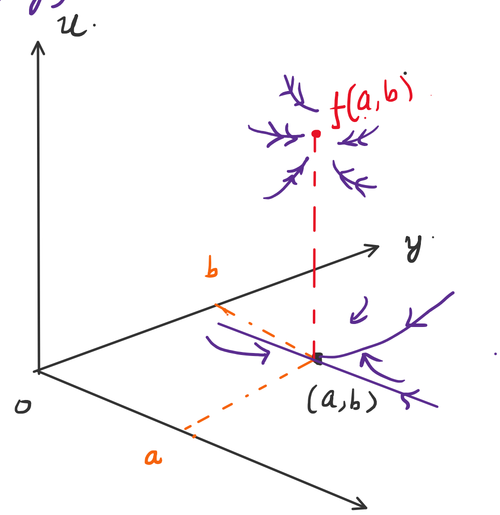
∴ LHL: $\lim_{x\to 0^-} \dfrac{1}{x} \to -\infty$.

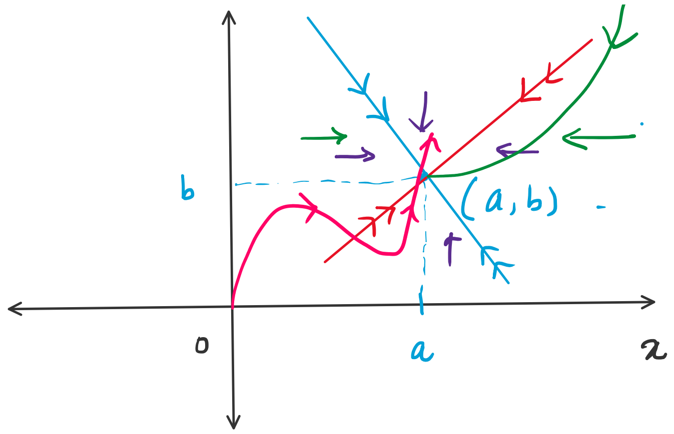
RHL: $\lim_{x\to 0^+} \dfrac{1}{x} \to +\infty$.

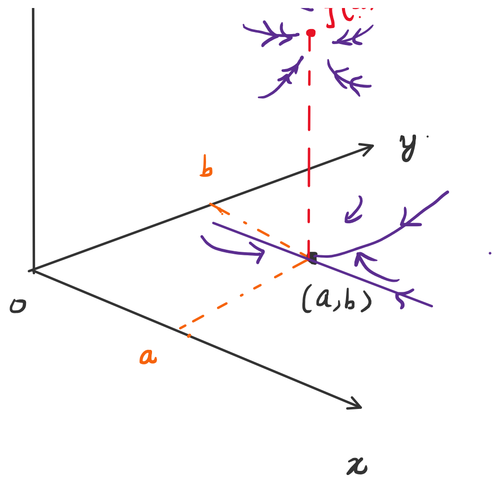
As LHL ≠ RHL ⇒ Limit does not exist at $x = 0$.

Eg: Consider $u = f(x, y)$. Check if $f(x, y)$ has limit at (a, b).







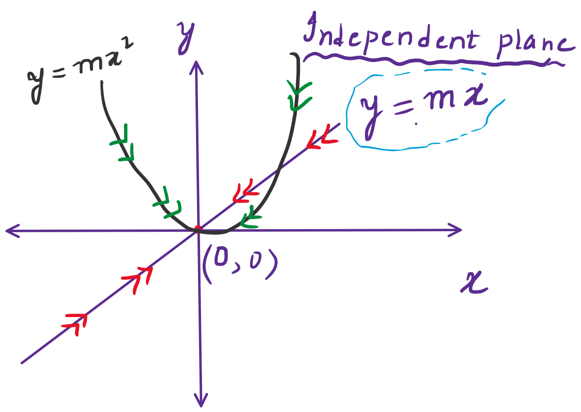


$$\lim_{(x,y)\to(a,b)} f(x,y)$$ will exist when no matter which path is taken on the independent x-y plane to arrive at pt (a,b), the functional value always tends to a same value $= l$.

Then only limit exists for the fn $f(x,y)$ at pt (a,b) and limiting value $= l$., i.e $\lim_{(x,y)\to(a,b)} f(x,y) = l$.

Q. Prove that $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$ does not exist.

Note: 2 common choice of paths: $\boxed{y = mx}$ / $y = mx^2$



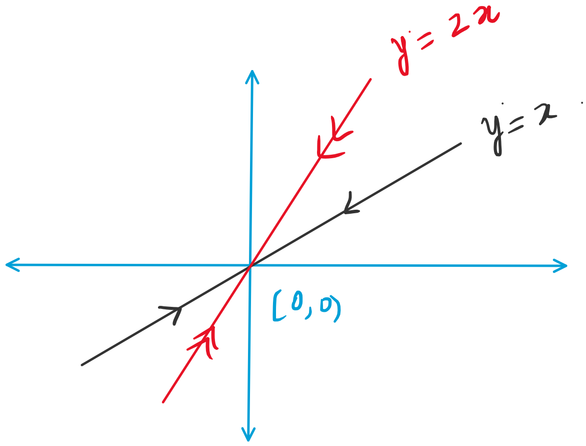
Consider the path $y = mx$.

as $x \to 0$, $y \to 0$.

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{\underbrace{(x,mx)\to(0,0)}} f(x,mx)$$

$$= \lim_{x\to 0} f(x,mx) = \lim_{x\to 0} \frac{x(mx)}{x^2+m^2x^2}.$$

$$= \lim_{x \to 0} f(x, mx) = \lim_{x \to 0} \frac{x(mx)}{x^2 + m^2x^2}.$$



$$= \lim_{x \to 0} \frac{mx^2}{x^2+m^2x^2} = \left(\frac{m}{1+m^2}\right)$$

varies with choice of m. Limit does not exists.

$y = mx$

(i) $m = 1 \Rightarrow l = 1/2$.

(ii) $m = 2 \Rightarrow l = \frac{2}{5}$