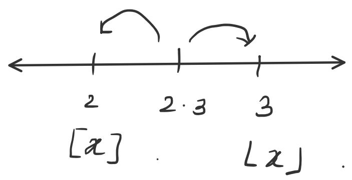
Domain and Range of a Function :-

(i) $f(x) = [x] \Rightarrow$ Greatest integer fn.

(ii) $f(x) = \lfloor x \rfloor \Rightarrow$ Least integer fn.



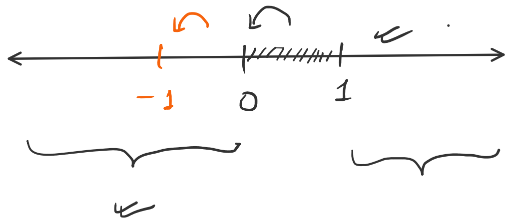
(iii) Let $f(x)$ and $g(x)$ be 2 fns &

define: $h(x) = f(x) \pm g(x)$

$\therefore$ Domain of $h(x) = D_f \cap D_g$.

Q. Find the domain of $f(x) = \dfrac{1}{[x]}$, $f: \mathbb{R} \to \mathbb{R}$

Rule out all values of x s.t $[x] = 0$.



$x = 0, \; [0] = 0$

$0 < x < 1, \; [x] = 0$.

$x \geq 1, \; [x] \neq 0$.

$-1 < x < 0, \; [x] = -1$

$\therefore \quad D_f : (-\infty, 0) \cup [1, \infty)$

Q. Find domain for $f: \mathbb{R} \to \mathbb{R}$ s.t $f(x) = \dfrac{1}{\sqrt{x - [x]}}$.

$x - [x] > 0$.

If x is an integer, $x = [x]$, $f(x) \to \infty$, Eliminate all $\mathbb{Z}$.

$\forall x \notin \mathbb{Z}, \; x - [x] > 0$.

$\therefore \quad D_f : \mathbb{R} \setminus \mathbb{Z}$ or $\mathbb{R} - \mathbb{Z}$.

Q. Find domain $f: \mathbb{R} \to \mathbb{R}$ s.t $f(x) = \dfrac{1}{\sqrt{[x]^2 - [x] - 6}}$

(a) $(-\infty, -2) \cup (9, \infty)$ (c) $(-\infty, -2) \cup [4, \infty)$

(a) $(-\infty,-2)\cup(9,\infty)$

(b) $(-\infty,-2)\cup[9,\infty)$

~~(c)~~ $(-\infty,-2)\cup[4,\infty)$

(d) $(-\infty,-2]\cup(4,\infty)$

Start with: $[x]^2-[x]-6>0$ .... (Quad in $[x]$)

Let $[x]=y$ $\Rightarrow$ $y^2-y-6>0$.

$\Rightarrow$ $(y+2)(y-3)>0$.

$\Rightarrow$ $y<-2$ or $y>3$.

$\Rightarrow$ $[x]<-2$ or $[x]>3$.

$x<-2$ or $x\geq 4$.

$\therefore$ $x\in(-\infty,-2)\cup[4,\infty)$

8. Find the domain $f:\mathbb{R}\to\mathbb{R}$ s.t $f(x)=\left(\sqrt{|[|x|-2]|-3}\right)^{-1}$

(a) $[-6,6]$

(b) $(-\infty,-6)\cup(6,\infty)$

~~(c)~~ $(-\infty,-6]\cup[6,\infty)$

(d) None.

$f(x)=\frac{1}{\sqrt{\cdots}}$

$k\in\mathbb{Z}^+$

$[x+k]=[x]+k$

$|[|x|-2]|-3>0$.

$|[|x|-2]|>3$. $\Rightarrow$ $|y|>3$ $\Rightarrow$ Either $y<-3$ or $y>3$.

Case I: $\underbrace{[|x|-2]}_{y}>3$. $\Rightarrow$ $[|x|-2]>3$.

$\Rightarrow$ $|x|-2\geq 4$.

$\Rightarrow$ $|x|\geq 6$.

$\Rightarrow$ $x\leq -6$ or $x\geq 6$

Case II: $[|x|-2]<-3$. $\Rightarrow$ $|x|-2<-3$.

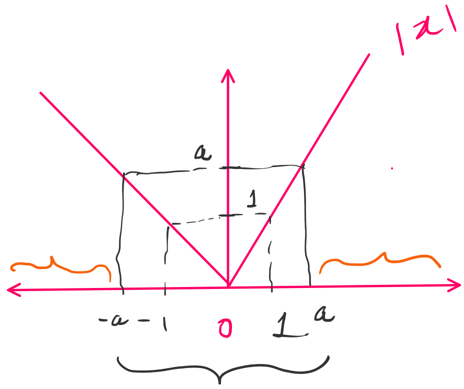
$\Rightarrow$ $|x|<-1$ X (Not possible)

D_f: $(-\infty,-6]\cup[6,\infty)$

$|x|$

$D_f : (-\infty, -6] \cup [6, \infty)$

Note: $f(x) = |x|$.



Find x s.t $\boxed{|x| < a} \Rightarrow -a < x < a$

Find x s.t $\boxed{|x| > a} \Rightarrow$

Either $x < -a$ or $x > a$.

Q. Find the domain of $f(x) = \dfrac{\log_2 (x+3)}{x^2 + 3x + 2}$.

(a) $\mathbb{R} - \{-1, -2\}$

(b) $(-2, \infty) - \{-1, -2\}$

(c) $(-3, \infty) - \{-1, -2\}$ ✓

(d) None.

$x + 3 > 0 \Rightarrow x > -3$

$x^2 + 3x + 2 \neq 0$.

$x \notin \{-1, -2\}$.

$\therefore$ Combining: $D_f : (-3, \infty) - \{-1, -2\}$.

Q. Find the range for $f : \mathbb{R} \to \mathbb{R}$ s.t $f(x) = \dfrac{x}{1+x^2}$

Let $y = \dfrac{x}{1+x^2}$, R_f : all possible values of y

$y(1 + x^2) = x$

$y x^2 - x + y = 0$ ..... [Quadratic in x]

$x = \dfrac{1 \pm \sqrt{1 - 4y^2}}{2y}$.

$f : \mathbb{R} \to \mathbb{R}$.

$x \in \mathbb{R}$, $y \in \mathbb{R}$.

$y = 0$, x becomes indeterminant $\Rightarrow \boxed{y \neq 0}$,

& $1 - 4y^2 \geq 0$.

$y = 0$, x becomes indeterminant $\Rightarrow \boxed{y \neq 0}$

& $1 - 4y^2 \geq 0$.

$(1-2y)(1+2y) \geq 0$.

$(2y-1)(2y+1) \leq 0$

$(y+\frac{1}{2})(y-\frac{1}{2}) \leq 0 \Rightarrow y \in [-\frac{1}{2}, \frac{1}{2}]$.

$\therefore R_f : [-\frac{1}{2}, \frac{1}{2}] - \{0\}$ or $[-\frac{1}{2}, 0) \cup (0, \frac{1}{2}]$.

HW.

Q. Let $f : \mathbb{R} - \mathbb{Z}$ such that $f(x) = \lceil x \rceil$ (Least Integer fn)

Define: $g(x) = |f(x)| - f(|x|)$. Then R_g:

(a) $\{0, 1\}$ (c) $\{-1, 0, 1\}$

(b) $[-1, 1]$ (d) $\{-1, 0\}$.