04 January 2024 17:03

Function of functions (Composite functions).

- Algebraic (Polynomials)
- Trigonometric.
- Logarithmic
- Exponentials.

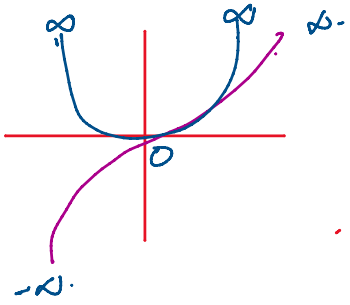
x^n (n is an integer) $\quad \sin x. \quad \log x. \quad e^x.$

$f(x) = x^n \qquad g(x) = \sin x.$

Domain $(-\infty, \infty) \qquad (-\infty, \infty)$

Range. $(-\infty, \infty)$ for odd n. $\quad [-1, 1]$

$[0, \infty)$ for even n.



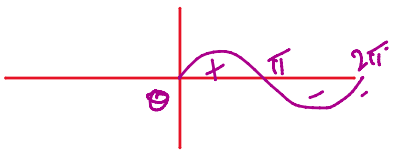
n is an integer $\quad f[g(x)] = f \circ g(x) = [g(x)]^n.$

$= (\sin x)^n = \sin^n x.$

Domain $(-\infty, \infty)$

Range. $[-1, 1] \rightarrow \boxed{n \text{ is odd}}$

$[0, 1] \rightarrow$ n is even



Domain of $g(x)$ is $[0, 2\pi]$ $\qquad g(x) = \sin x$

Range of $f \circ g(x) \rightarrow [-1, 1]$ for odd n.

$[0, 1]$ for even n.

$y = a^n$ power $\rightarrow$ integer. fraction.

base $\rightarrow$ +ve, -ve. +ve

not always true

$a = -1 \qquad a^{1/2} \qquad a^{1/3} \qquad (-1)^3 = -1$

$(-1)^{1/2} = \qquad (-1)^{1/3} = \qquad -1 = (-1)^{1/3}$

undefined

$f(x) = x^n \qquad g(x) = \sin x.$

$f(x) = x^n$ $\qquad$ $g(x) = \sin x$.

n is an integer

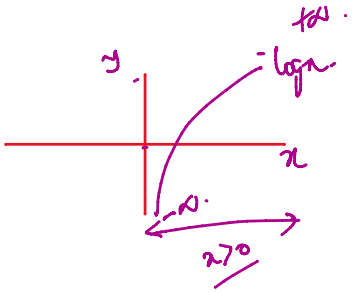
$g[f(x)] = g \circ f(x) = \sin(x^n)$

	n odd	n even
	$n = 3$.	$n = 2$.
$g \circ f(x) =$	$\sin(x^3)$	$\sin(x^2)$
Domain	$(-\infty, \infty)$	$(-\infty, \infty)$
Range	$[-1, 1]$	$[-1, 1]$

$x = (-\pi, \pi)$

$\pi = 3.14$ $\quad 2\pi \approx 6.28$ $\quad x^2 = (0, \pi^2) = (0, 10) \Rightarrow (0, 2\pi)$

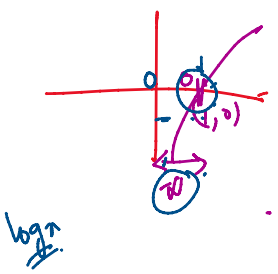
$\pi^2 \approx 10$.



$f(x) = \log x$ $\qquad$ $g(x) = x^n$, n is an integer.

$f \circ g(x) = n \log x$ $\qquad$ $g \circ f(x) = (\log x)^n$, n is odd or even

Domain	$(0, \infty)$	$(0, \infty)$	
Range	$(-\infty, \infty)$	$(-\infty, \infty)$	n is odd
		$[0, \infty)$	n is even.

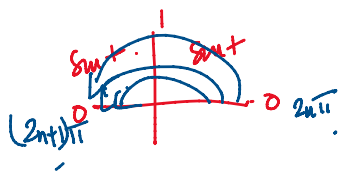


$f(x) = \log x$. $\qquad$ $g(x) = \sin x$.

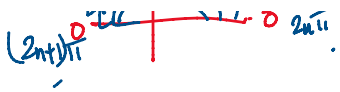
$f \circ g(x) = \log(\sin x)$ $\qquad$ $g \circ f(x) = \sin(\log x)$

$[0, 1]$

Domain	$[2n\pi, (2n+1)\pi]$ $\cancel{[-1, 1]}$	$(0, \infty)$
Range	$(-\infty, 0]$	$[-1, 1]$



$0 - \pi$, $2\pi - 3\pi$, $4\pi - 5\pi$



$y = \dfrac{x^2+x+1}{x^2-3x+2}$

Domain = $(-\infty, \infty) - \{1, 2\}$

Range =

$D = 1$

$(x-1)(x-2)$

$x \neq 1, 2$

$yx^2 - 3yx + 2y = x^2 + x + 1$

$yx^2 - x^2 - 3yx - x + 2y - 1 = 0$

$(y-1)x^2 - (3y+1)x + (2y-1) = 0$

$D \geq 0$

$(3y+1)^2 \geq 4(y-1)(2y-1)$

$9y^2 + 6y + 1 \geq 4(2y^2 - 3y + 1)$

$y^2 + 18y - 3 \geq 0.$

$(y-\alpha)(y-\beta) \geq 0.$

$y \geq \beta$ or $y \leq \alpha$

$(-\infty, \alpha] \cup [\beta, \infty)$

$D > 0$

$y^2 + 18y - 3$

$y = \dfrac{-18 \pm \sqrt{324+12}}{2}$

$= \dfrac{-18 \pm \sqrt{336}}{2} = \alpha, \beta$

$\alpha = \dfrac{-18 - \sqrt{336}}{2}$ $\beta = \dfrac{-18 + \sqrt{336}}{2}$