Heteroscedasticity.

↓

violation of assumption of homoscedasticity

$Y_i = \alpha + \beta x_i + u_i$

$E(u_i^2) \neq \sigma^2$ (const)

ie, variance of random error (u_i) is non constant

↓

heteroscedasticity.

but $E(u_i^2) = \sigma_i^2$ for all $i = 1(1)n$

Here $\hat{\alpha}$ and $\hat{\beta}$ (In presence of heteroscedasticity) it does not satisfy BLUE ⇒ Best Linear Unbiased Estimator

[In presence of heteroscedasticity OLS estimates of $\hat{\alpha}$ and $\hat{\beta}$ is unbiased but do not satisfy the condition for min variance.

Hence OLS cannot be the best estimate process] it does not follow BLUE.

Therefore we can use the **Method of Generalised Least Square (GLS) method:**

Unfortunately the usual OLS method does not follow this strategy and therefore does not make use of the information contained in the unequal variability of dependent variable Y

In this case we will assign equal weights or importance to each observation and the method is known as GLS. It is capable of producing estimators that are BLUE.

Model : $Y_i = \beta_1 + \beta_2 X_i + u_i$ ①

Let $X_{0i} = 1$, then $Y_i = \beta_1 X_{0i} + \beta_2 X_i + u_i$

Now assume that the heteroscedastic variance σ_i^2 is known.

On dividing σ_i on both sides of eq ①, we have,

$$\frac{Y_i}{\sigma_i} = \beta_1 \left(\frac{X_{0i}}{\sigma_i}\right) + \beta_2 \left(\frac{X_i}{\sigma_i}\right) + \left(\frac{u_i}{\sigma_i}\right)$$

ie, $$\boxed{Y_i^* = \beta_1^* X_{0i}^* + \beta_2^* X_i^* + u_i^*}$$

What is the purpose of transforming the original model?

$$Var(v_i^*) = E(u_i^*)^2 = E\left(\frac{u_i}{\sigma_i}\right)^2$$

$$= \frac{1}{\sigma_i^2} E(u_i^2)$$

$$= \frac{1}{\sigma_i^2} \times \sigma_i^2 = 1$$

$$= \text{const}$$

That is variance of the transformed disturbance term is v_i^* which is homoscedastic.

So to conclude the estimated β_1^* and β_2^* are now BLUE and not the OLS estimators $\hat{\beta}_1$ and $\hat{\beta}_2$

White's General Heteroscedasticity Test:

$$Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + u_i$$

1. we estimate and obtain residuals, $\hat{u}_i$.
2. we run the following auxiliary regression

② We run the following auxiliary regression

$$\hat{u}_i = \alpha_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_2^2 + \alpha_5 X_3^2 + \alpha_6 X_2 X_3 + V_i$$

we will find out/obtain R^2 from this auxiliary regression.

③ H_0: No heteroscedasticity, under H_0 it can be shown that sample size n time R^2 obtained from auxiliary regression follows χ^2 distribution with df equal to no of regressors (excluding the const term) in the auxiliary regression.

ie $\quad n \cdot R^2 \sim \chi^2_{df}$ ① $\quad$ (in our example there are 5 regressors in auxiliary regression)

Step 4: If the chi-square $(n \cdot R^2)$ value in eq ① is (a) greater than critical chi-square value at the chosen level of significance the conclusion is that there is heteroscedasticity ie H_0 is rejected. Look step ③.

ie H_0 is rejected.

(b) And if it does not exceed critical chi-square value $\Rightarrow$ NO heteroscedasticity $\Rightarrow$ H_0 is accepted.