Real Analysis

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Sets & Functions

For exam no of subsets $\rightarrow 2^n$

Schrodinger - Bernstein Theorem

A & B $f: A \to B$ $g: B \to A$

MAP MAP

Exam special theory session Then Bijection injective Map $\boxed{h: A \to B}$

Counting of functions

A, B non-empty sets

element $|A| = 2, 3, 4$ $|A| = 3$ then

$|A| = m$, $|B| = n$ $f: A \to B$ function

$\boxed{\text{Case I}}$ if $m > n$

total onto functions are $n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - {}^nC_3(n-3)^m$ - - - ..

$f: A \to B$

one-one $\rightarrow 0$

Bijection $\rightarrow 0$.

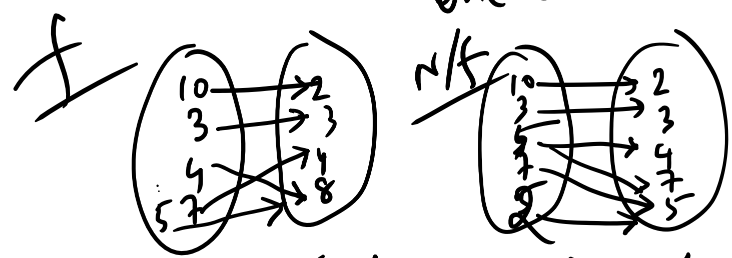
II $m < n$ onto $\rightarrow 0$

one-one $\rightarrow {}^nC_m(m!)$

Bijection $\rightarrow 0$

One-one → ...

Bijection → 0

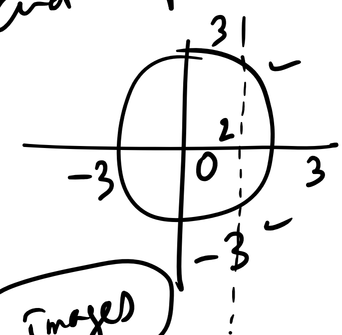


Every function is a Relation

But not every relation is a function

Well-defined → elements are mapped to a unique and specific image correspondingly

$x^2+y^2=3^2$



$f(x)=2x$

$f(1)=2, -2$ ×

@ $x=2$ 2 Images

m = n

Onto → $n!$

10, 10 10 10 --- 10

n^m 10^{10}

Bijection → $n!$

One-one → $n!$

Total functions $n^m = |B|^{|A|}$

$f(x)=2^{x^4}$

$=2x^2$

$=\pm 2\sqrt{x}$

New Zealand

7 digit 850 crore

10^{10} 1000 crore

9836793

98367930

983679307

9836793076

= 120

9836793067

9836793076

TIFR GS. Math

TIF

Sequence & Series ..

Every Conv → Bounded

may not be true

1.7.2AM

NAT

Ex $\{a_n\} = (-1)^n$

GLB → -1 → Inf

LUB → $+1$ → Sup

Highest → Superior → L

Inferior

KFC

Hut & company

$85 \times 2 = \boxed{170}$

$\boxed{185}$

H & L

Sup → LUB

Inf → GLB

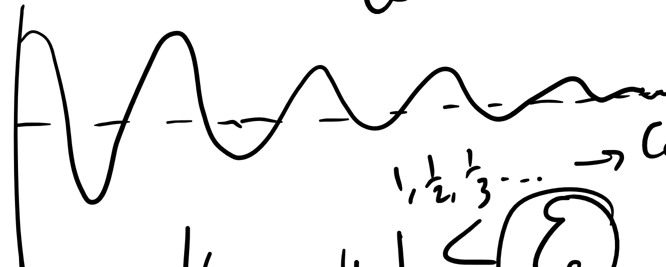
-1, $+1$ non Conv

$(-1)^n$ $-1, 1, -1, 1, -1, 1 \ldots$

$\frac{1}{100}, \frac{1}{101} \ldots \frac{1}{\infty} \to 0$

(ii) Every Conv → Cauchy Sequence.

Converse may not be True..



XI - XII

$1, \frac{1}{2}, \frac{1}{3} \ldots$ → Cauchy Sequence ..

$|u_{m+r} - u_n| < \epsilon$ $\forall\, n, r \geq m_0$ ..

u_{m_1} u_{m_2} u_{m_3}

1, 0.5, 0.33, 0.25, 0.20

$$1,\ 0.5,\ 0.33,\ 0.25,\ 0.20$$

0.5   0.17   0.08   0.05

1, 1.5, 1.4, 1.3,

$$\frac{1}{1},\ \frac{3}{2},\ \frac{7}{5},\ \frac{15}{11},\ \frac{31}{20}$$

+1 +3 +6 +9   +4 +8 +16

Converges to arrational number

not a Cauchy

Because there is no irrational number

Monotonic vs Strictly monotonic

$U_n > U_m$

$U_n \geq U_m$

Concentration   Tv-movie

The wonderful story of Henry Sugar

2/3

Dev Patel

Ramanujan

Sir Benkinsy 27

slumdog million...

Comparison Test

$\sum a_n$ → positive term series

If $a_n \leq c_n$ for $n \geq N_0$ N_0 → some fixed integer

If $\sum c_n$ converges and then a_n converges

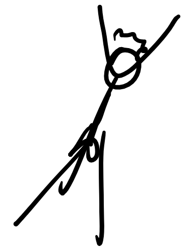
(ii) If $a_n \geq d_n \geq 0$ for $n \geq N_0$ $\sum d_n$ diverges

the $\sum a_n$ diverges

Pass

(ii) If u_n ... the

Pass



10 ✓
9 ✓
8 ✓
7 ✓
. .
. .
1 ✓

$$\alpha = \limsup_{n\to\infty} \sqrt[n]{a_n}$$

$$\sum a_n \to \begin{cases} \text{Con} & \alpha < 1 \\ \text{Div} & \alpha > 1 \\ \text{Nothing} & \alpha = 1 \end{cases}$$

M-test .. $\{f_n\}$ is a sequence defined on E which converges to $f(x)$ on E.

If $M_n = \sup |f_n(x) - f(x)| \longrightarrow$ LUB

$\{f_n\} \to f$ uniformly on E iff $M_n \to 0$ as $n \to \infty$

#

Failure of all tests

Gauss Test

$$1 + \frac{2^2}{3^2} + \frac{2^2 4^2}{3^2 5^2} + \frac{2^2 4^2 6^2}{3^2 5^2 7^2} + \dots \infty$$

$$u_n = \frac{2^2 4^2 6^2 \dots (2n)^2}{3^2 5^2 7^2 \dots (2n+1)^2}$$

$$u_{n+1} \to \frac{\dots (2n)^2 (2n+2)^2}{\dots (2n+1)^2 (2n+3)^2}$$

$$u_{n+1} \to \frac{- \; - \; \cdot (2n)\, a \; \cdot \; \cdot}{- \; - \; - \; \cdot (2n+1)^2 (2n+3)^2}$$

$$\frac{u_n}{u_{n+1}} \to \frac{(2n+3)^2}{(2n+2)^2} = \frac{(1+3/2n)^2}{(1+1/n)^2}$$

$$\frac{1}{1} \to 1 \text{ Test fails}$$

Raabe's test

$$\frac{u_n}{u_{n+1}} = \frac{(1+3/2n)^2}{(1+1/n)^2} = \left(1+\frac{3}{2n}\right)^2 \left(1+\frac{1}{n}\right)^{-2}$$

$$(1+x)^{-2} = 1 - 2x + 3x^2 - \cdots$$
$$= 1 - 2x + O(x^2)$$

$$\left(1+\frac{3}{2n}\right)^2 \left(1+\frac{1}{n}\right)^{-2}$$

$$= \left(1+\frac{9}{4n^2}+\frac{3}{n}\right)\left(1-\frac{2}{n}+\frac{3}{n^2}-\cdots\right)$$

$$= 1 + \frac{9}{4n^2} + \frac{3}{n} - \frac{2}{n} - \frac{3}{n^2} - \frac{6}{n^2} + O\left(\frac{1}{n^2}\right)$$

$$= 1 - \frac{2}{n} + \frac{3}{n} + O\left(\frac{1}{n^2}\right)$$

$$= 1 + \frac{1}{n}$$

$$\boxed{\text{Gauss Test}}$$

$$\left|\frac{u_n}{u_{n+1}}\right| = 1 + \frac{h}{n} + \frac{B(n)}{n^r}$$

$r > 1$

Conv. for $\boxed{h > 1}$

divr. $\boxed{h \le 1}$

$$1 - \frac{h_1}{n} + \frac{B(n)}{n^r}$$

Conv. Absolutely iff $\boxed{h_1 > 1}$

$1 - \frac{h_1}{n} + \frac{r_3(n)}{n^2}$

Special Case is n-MATH Interview

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots$

$$\frac{u_n}{u_{n+1}} = \left(1+\frac{1}{n}\right)^p \left(1+\frac{2}{n}\right)^{-p}$$

$$\Rightarrow \left(1 + p\cdot\frac{1}{n} + \frac{p(p-1)}{2!}\frac{1}{n^2} + \dots\right)\left(1 - \frac{p\cdot 1}{2n} + \frac{p(p+1)}{2!}\frac{1}{n^2} - \dots\right)$$

$$\Rightarrow \left(1 + \frac{p}{n} - \frac{p}{2n} + O\left(\frac{1}{n^2}\right)\right) + O\left(\frac{1}{n^2}\right)$$

$$\Rightarrow 1 + \frac{p}{2n} + O\left(\frac{1}{n^2}\right)$$

$$\Rightarrow 1 + \frac{p/2}{n}$$

$$\boxed{h \to p/2}$$

If $p > 2$ or $p \leq 2$

In case power series the sanguineness of the Gauss Test is not applicable..

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