Nature of the Goods:

∴ For utility maximization:

$$\max_{\{x_1, x_2\}} u = u(x_1, x_2) \quad \text{subject to} \quad M = P_1 x_1 + P_2 x_2$$

on solving, we get the Marshallian demand curves:

$$x_1^* = x_1^*(P_1, P_2, M) \qquad x_2^* = x_2^*(P_1, P_2, M)$$

i) Based on Money Income (M): [Eg: Good 1]

$\frac{\partial x_1^*}{\partial M} > 0 \Rightarrow$ Normal Good. [$e_{1M} > 0$]

$\frac{\partial x_1^*}{\partial M} < 0 \Rightarrow$ Inferior Good. [$e_{1M} < 0$]

$\frac{\partial x_1^*}{\partial M} = 0 \Rightarrow$ Necessary Good. [$e_{1M} = 0$]

ii) Based on Own Price (P_1): [Eg: Good 1]

$\frac{\partial x_1^*}{\partial P_1} < 0 \Rightarrow$ Normal Good [Follows the Law of demand] [$e_{11} < 0$]

$\frac{\partial x_1^*}{\partial P_1} > 0 \Rightarrow$ Giffen Good [Violates the law of demand] [$e_{11} > 0$]

$\frac{\partial x_1^*}{\partial P_1} = 0 \Rightarrow$ No specific name [not empirically true] [$e_{11} = 0$]

iii) Based on Cross Price (P_2): [Eg: Good 1]

$\frac{\partial x_1^*}{\partial P_2} > 0 \Rightarrow P_2 \uparrow \Rightarrow x_2 \downarrow \Rightarrow x_1 \uparrow \Rightarrow$ Goods are substitutes [$e_{12} > 0$]

$\frac{\partial x_1^*}{\partial P_2} < 0 \Rightarrow P_2 \uparrow \Rightarrow x_2 \downarrow \Rightarrow x_1 \downarrow \Rightarrow$ Goods are complements. [$e_{12} < 0$]

(*) Elasticity of demand $= \frac{\% \Delta Q}{\% \Delta P} \gtrless 0 \Rightarrow \frac{dQ/Q \times 100}{dP/P \times 100} = \left(\frac{dQ}{dP} \cdot \frac{P}{Q}\right)$

(*) Elasticity of demand $= \dfrac{\%\Delta Q}{\%\Delta P} \gtrless 0 \Rightarrow \dfrac{dQ/Q \times \cancel{100}}{dP/P \times \cancel{100}} = \left(\dfrac{dQ}{dP}\cdot\dfrac{P}{Q}\right)$

Denote: e_{12} = Elasticity of Good 1 w.r.t Price of Good 2.

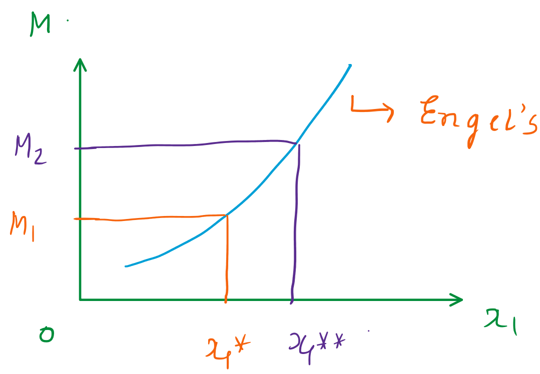
e_{11} = Elasticity of Good 1 w.r.t Price of Good 1

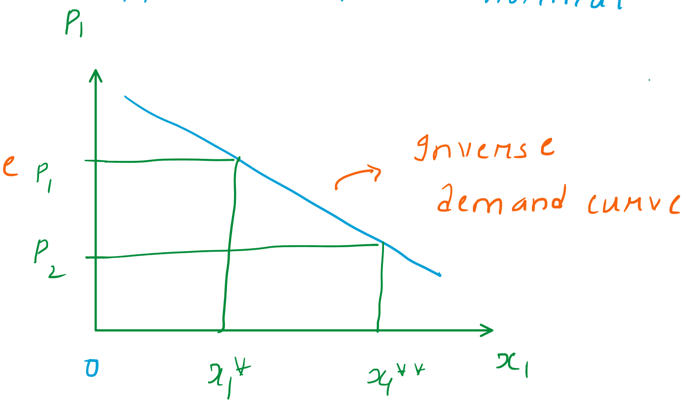
e_{1M} = Elasticity of Good 1 w.r.t Money Income.

$$e_{11} = \frac{\partial x_1}{\partial P_1}\cdot\overset{>0}{\left(\frac{P_1}{x_1}\right)} \quad ; \quad e_{12} = \frac{\partial x_1}{\partial P_2}\cdot\overset{>0}{\left(\frac{P_2}{x_1}\right)} \quad ; \quad e_{1M} = \frac{\partial x_1}{\partial M}\overset{>0}{\left(\frac{M}{x_1}\right)}$$

Note: sign of e_{11}, e_{12}, e_{1M} depend upon the sign of the derivatives.

We have: $\boxed{x_1^* = x_1^*(\overset{-}{P_1}, P_2, \overset{+}{M})}$ ⇒ Suppose the Good is normal.





Engel's Curve: Locus of (M, Q) s.t utility is maximized.

Inverse Demand Curve: Locus of (P, Q) s.t utility is maximized.

Note: (i) Inferior Good, engel curve is -vely sloped.

(ii) For a Giffen Good, demand curve is positively sloped.

Properties of Marshallian Demand Curves:

Marshallian demand curves: $x_1^* = x_1^*(M, P_1, P_2)$ ---- (i)

$x_2^* = x_2^*(M, P_1, P_2)$

Q. Suppose that M, P_1, P_2 increases in the same proportion. Find the effect on the demands for Good 1 & Good 2.

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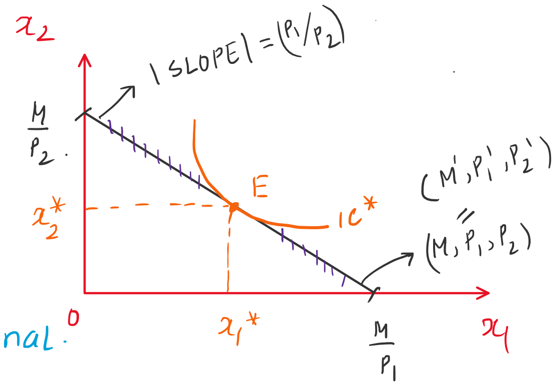
say, now, $M' = k \cdot M, \; k > 1$.
$P_1' = k P_1, \; k > 1$
$P_2' = k P_2, \; k > 1$.



Initially: $(M, P_1, P_2) \longrightarrow (M', P_1', P_2')$: Final.

Initial B.L : $M = P_1 x_1 + P_2 x_2$.

Final B.L : $M' = P_1' x_1 + P_2' x_2$.

$$kM = kP_1 x_1 + kP_2 x_2$$

$$\cancel{k}M = \cancel{k}[P_1 x_1 + P_2 x_2]$$

$\boxed{M = P_1 x_1 + P_2 x_2} \Rightarrow$ same as the old B.L [B.L remains unchanged]

Indifference curves are constructed from the utility fn: $u = u(x_1, x_2)$: IC's also do not change.

$\therefore$ BL unchanged, IC's unchanged $\Rightarrow$ demand pts also unchanged.

$$\begin{cases} x_1^*(M, P_1, P_2) = x_1^*(kM, kP_1, kP_2) \\ x_2^*(M, P_1, P_2) = x_2^*(kM, kP_1, kP_2) \end{cases}$$

If money income and all prices are changed in the same proportion, then demands are unchanged.

$\therefore$ Demand fns are homogeneous of degree zero in Income, prices.

Q. Suppose demand for Good 1 is given by: $x_1^* = P_1^{\alpha} P_2^{\beta} M^{\gamma}$. What should be restriction on α, β, γ s.t this a valid demand fn.

Suppose P_1, P_2, M are changed in the same prop k.

Initial: (P_1, P_2, M) $\quad$ Final (kP_1, kP_2, kM)

$\hookrightarrow x_1^*$ $\quad$ $\hookrightarrow x_1^{**}$

$\hookrightarrow x_1^*$ $\hookrightarrow x_1^{**}$

$\therefore$ Initially: $x_1^* = P_1^{\alpha} P_2^{\beta} M^{\gamma}$

Final: $x_1^{**} = (kP_1)^{\alpha} (kP_2)^{\beta} (kM)^{\gamma} = k^{\alpha+\beta+\gamma} \cdot P_1^{\alpha} P_2^{\beta} M^{\gamma}$.

By property of Marshallian demand:

$$x_1^* = x_1^{**}$$

HW.