12 December 2023 18:02

Triple Product.

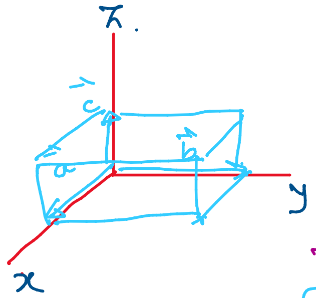
$[\vec{a}\ \vec{b}\ \vec{c}] \to$ If the vol of the box $= 0$

$\Downarrow$

there is no box formed by $\vec{a}, \vec{b}, \vec{c}$

$\Downarrow$

$\boxed{\vec{a}, \vec{b}, \vec{c} \text{ are co-planar.}}$



$\vec{a} = 2\hat{i}$

$\vec{b} = 3\hat{j}$

$\vec{c} = 4\hat{k}.$

$\vec{b} \times \vec{c} = 12\hat{i}.$

$\boxed{(\vec{b} \times \vec{c}) \cdot \vec{a}} = 24$

$\vec{a} \times \vec{b} = 2\hat{i} \times 3\hat{j} = 6\hat{k}.$

$\boxed{(\vec{a} \times \vec{b}) \cdot \vec{c}} = 6\hat{k} \cdot 4\hat{k} = 24$

$(\vec{c} \times \vec{a}) = 8\hat{j}$ $\quad \boxed{(\vec{c} \times \vec{a}) \cdot \vec{b}} = 24.$

a b c

b c a

c a b

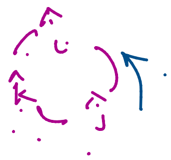
$[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$

$\times \cdot$ $\quad \times \cdot$ $\quad \times \cdot$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}.$



$\hat{j} \times \hat{i} = -\hat{k}$

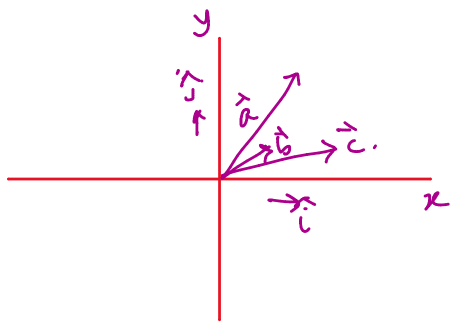
$\hat{i} \times \hat{k} = -\hat{j}$

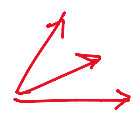
$\hat{k} \times \hat{j} = -\hat{i}$

$\vec{a} = 2\hat{i} + 3\hat{j}$ $\quad \vec{b} = +\hat{i} + 2\hat{j}$ $\quad \vec{c} = 3\hat{i} + \hat{j}.$

$\vec{a} \times \vec{b} = 4\hat{k} - 3\hat{k} = \hat{k}.$

$(\vec{a} \times \vec{b}) \cdot \vec{c} = \hat{k} \cdot (3\hat{i} + \hat{j}) = 0$





Limits

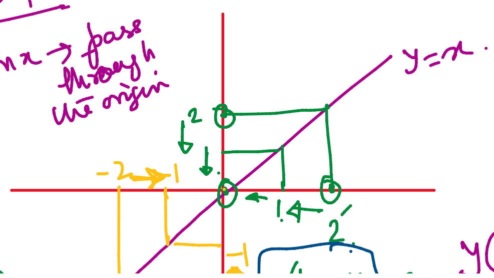
$\boxed{y = x}$ $\longleftrightarrow$ NOT SAME $\boxed{y = \frac{x^2}{x}.}$

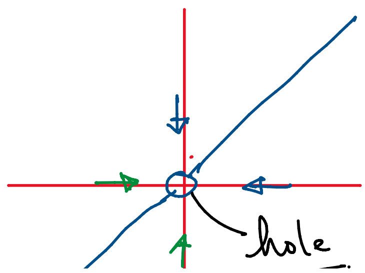
$y = x, \quad x \neq 0$

$= \text{undefined}, \ x = 0.$

$\underline{y = mx + c}$

$y = mx \to$ pass through the origin

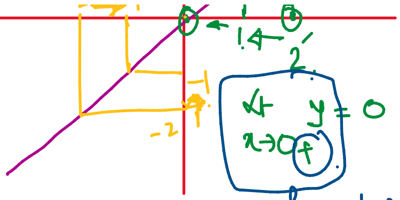




$y = \frac{x^{\cancel{2}}}{\cancel{x}}$

removing the discontinuity

$\Downarrow$



$\lim_{x\to 0^-} y = 0$ from -ve side — LHL

$\lim_{x\to 0^+} y = 0$ from +ve side — RHL

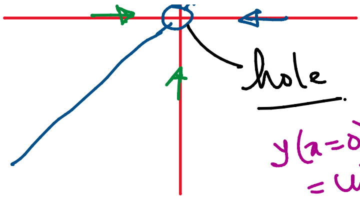
$y(x=0) = 0$

$$\boxed{\lim_{x\to 0^-} y = 0 = \lim_{x\to 0^+} y}$$

$\Downarrow$ $\lim_{x\to 0} y$ exists at $x=0$

$$\boxed{\lim_{x\to 0^-} y = \lim_{x\to 0^+} y = y(0)}$$

$\Downarrow$ y is continuous at $x=0$.



$y(x=0)$ $=$ ~~undefined~~

$\Downarrow$ $x=0$ is a removable discontinuity

$\Downarrow$ hole

$$\boxed{\lim_{x\to 0^-} y = 0 = \lim_{x\to 0^+} y}$$

$\Downarrow$ $\lim_{x\to 0} y$ exists at $x=0$

$$\boxed{\lim_{x\to 0^-} y = \lim_{x\to 0^+} y \neq y(0)}$$

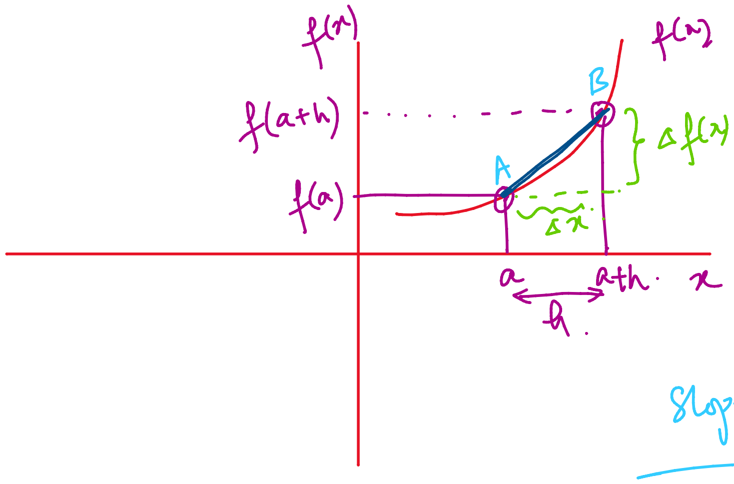
$\Downarrow$ y is discontinuous at $x=0$.

Condition for continuity

LHL = RHL = functional value.

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$$

Derivative

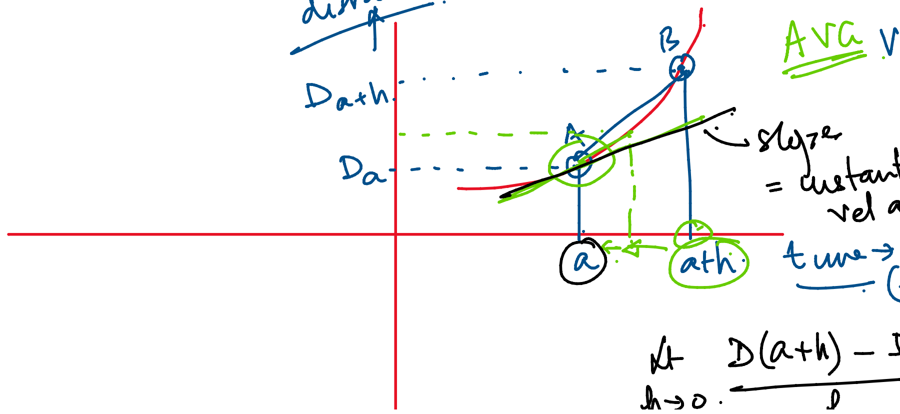


rate of change of $f(x)$ w.r.t x

$$= \frac{\text{change of } f(x)}{\text{change of } x} = \frac{\Delta f(x)}{\Delta x}$$

$$= \frac{f(a+h) - f(a)}{(a+h) - a}$$

Slope of AB $= \boxed{\dfrac{f(a+h) - f(a)}{h}}$



AVG vel between A and B

$$= \frac{D_{a+h} - D_a}{h}, \quad h \to 0$$

$$\lim_{h\to 0} \frac{D(a+h) - D(a)}{h} = \text{instantaneous vel at A.}$$

$$\lim_{h\to 0} \frac{D(a+h) - D(a)}{h} = \text{instantaneous vel at } A.$$

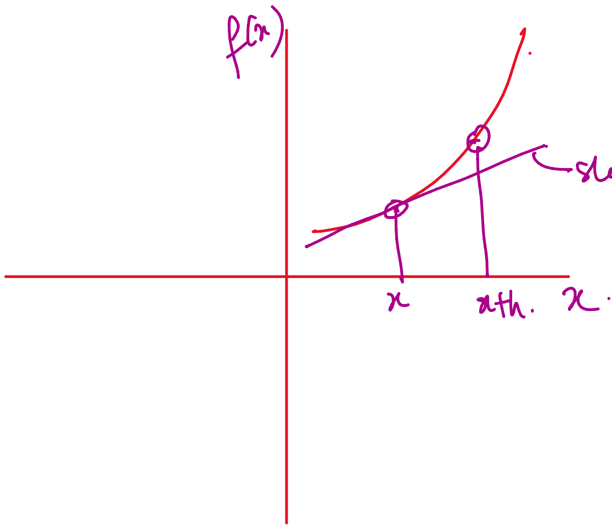
instantaneous rate of change.

↓

slope of the tangent at A.

⇓

derivative of $D(t)$ at A



slope = derivative of $f(x)$

↓

$$f'(x) = \frac{d}{dx} f(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}.$$