$S_1 = Y_1 - C_1$

Intertemporal Choice

Suppose we have a consumer living for 2 periods - Pd 1 & Pd 2.

Parameters: Y_1, Y_2.

Variables: C_1, S_1, C_2

Utility fn: $u(c_1, c_2)$; $\dfrac{\partial u}{\partial c_1} > 0, \dfrac{\partial u}{\partial c_2} > 0$

	Income	Consumption	Savings
Pd 1	Y_1	C_1	S_1
Pd 2	Y_2	C_2	–

Marginal utilities are +ve for both periods.
[C_1, C_2 are desirable].

Budget constraint: $C_2 = Y_2 + (1+r)S_1$ [r = rate of interest (real)]

$$C_2 = Y_2 + (1+r)[Y_1 - C_1].$$

$$\Rightarrow \boxed{(1+r)C_1 + C_2 = (1+r)Y_1 + Y_2}$$

↳ Intertemporal B.L.

Utility Maximization:

$$\max_{\{c_1, c_2\}} u = u(c_1, c_2) \quad \text{s.t} \quad (1+r)C_1 + C_2 = (1+r)Y_1 + Y_2$$

Solving: C_1^* [optimal consumption level in pd 1].
C_2^* [optimal consumption level in pd 1]

Note: If $C_1^* > Y_1 \Rightarrow$ Borrower.
If $C_1^* < Y_1 \Rightarrow$ Lender

Q. Suppose a consumer lives for 2 periods and consumes c_1, c_2 units in Pd 1, Pd 2. Utility f...

Q. Suppose a consumer lives for 2 periods and consumes c_1, c_2 units in Pd 1, Pd 2. Utility fn: $u(c_1, c_2) = \log c_1 + \frac{1}{1+\rho} \cdot \log c_2$

[$\rho > 0 \Rightarrow$ rate of discount].

(i) Find the optimal level of consumption in both pds.

(ii) Find the condition for $c_1^* > c_2^*$, $c_2^* > c_1^*$, $c_1^* = c_2^*$

(iii) Find the condition for consumer to be a borrower in Pd 1.

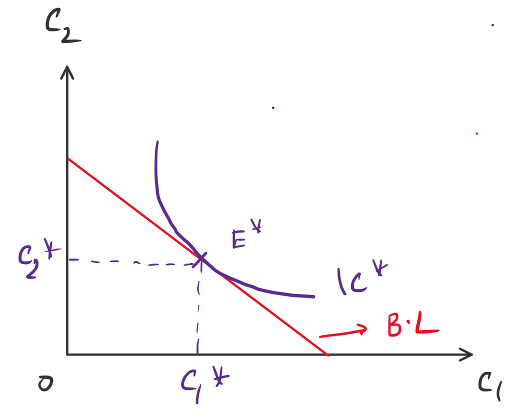
(i) $u(c_1, c_2) = \log c_1 + \frac{1}{1+\rho} \log c_2$

	Y	C	S
Pd 1	Y_1	C_1	S_1
Pd 2	Y_2	C_2	–

B·L: $C_2 = Y_2 + (1+r) S_1$

$C_2 = Y_2 + (1+r)[Y_1 - C_1]$

$$(1+r) Y_1 + Y_2 = (1+r) C_1 + C_2$$



At opt: Slope of IC = Slope of B·L.

B·L: $(1+r) Y_1 + Y_2 = \boxed{(1+r) C_1} + C_2$

Diff: $0 = (1+r)\, dC_1 + dC_2$.

$$\left.\frac{dC_2}{dC_1}\right|_{B\cdot L} = -(1+r)$$

IC: $u(c_1, c_2) = \log c_1 + \frac{1}{1+\rho} \cdot \log c_2$.

Diff: $du = \frac{1}{c_1} \cdot dc_1 + \frac{1}{1+\rho} \cdot \frac{1}{c_2} \cdot dc_2$

For IC: $du = 0 \Rightarrow 0 = \frac{1}{c_1} \cdot dc_1 + \frac{1}{1+\rho} \cdot \frac{1}{c_2} \cdot dc_2$

$$\left.\frac{dc_2}{dc_1}\right|_{IC} = -\frac{(1+\rho)\, c_2}{c_1}$$

ac₁ l'e c_1

At opt : $\quad \cancel{-}\,\dfrac{(1+\rho)\,c_2}{c_1} = \cancel{-}\,(1+r)$

$$\Rightarrow \boxed{(1+\rho)\,c_2 = (1+r)\,c_1} \Rightarrow \boxed{\frac{c_2^*}{c_1^*} = \left(\frac{1+r}{1+\rho}\right)}$$

B.L : $\quad (1+r)Y_1 + Y_2 = (1+r)\,c_1 + c_2$

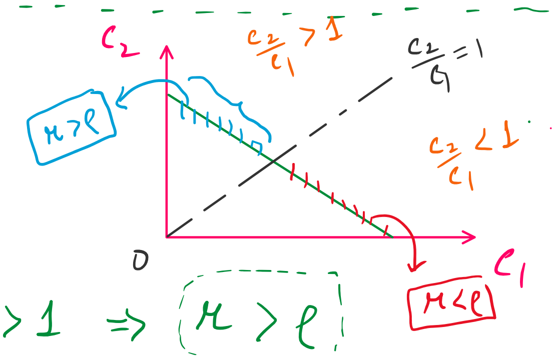
$(1+r)\,Y_1 + Y_2 = (1+\rho)\,c_2 + c_2$.

$(1+r)\,Y_1 + Y_2 = (\rho+2)\,c_2$

$$\boxed{c_2^* = \frac{1}{\rho+2}\left[(1+r)\,Y_1 + Y_2\right]}$$

$$\boxed{c_1^* = \frac{(1+\rho)\,c_2^*}{(1+r)} = \left(\frac{1+\rho}{1+r}\right)\cdot\frac{1}{\rho+2}\left[(1+r)\,Y_1 + Y_2\right]}$$

(ii) $\quad \dfrac{c_2^*}{c_1^*} = \left(\dfrac{1+r}{1+\rho}\right)$



For $\boxed{c_2^* > c_1^*} \Rightarrow \dfrac{c_2^*}{c_1^*} > 1 \Rightarrow \left(\dfrac{1+r}{1+\rho}\right) > 1 \Rightarrow \boxed{r > \rho}$

↳ [Bias towards future consumption] .

For $c_2^* < c_1^* \Rightarrow \boxed{r < \rho} \rightarrow$ [Bias towards present consumption]

for $c_1^* = c_2^* \Rightarrow \boxed{r = \rho} \rightarrow$ [consumption smoothening] .

(iii) For borrower : $\quad c_1^* > Y_1$

HW : $\quad \left(\dfrac{1+\rho}{1+r}\right)\cdot\dfrac{1}{\rho+2}\left[(1+r)\,Y_1 + Y_2\right] > Y_1$