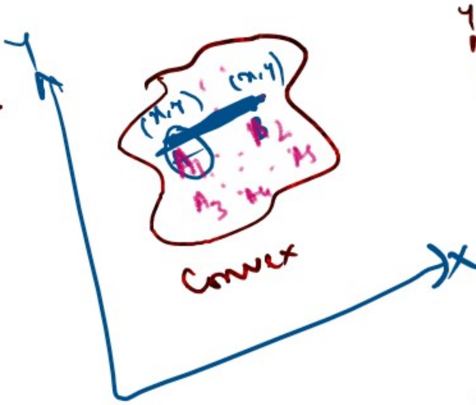
10 August 2023 08:04 PM

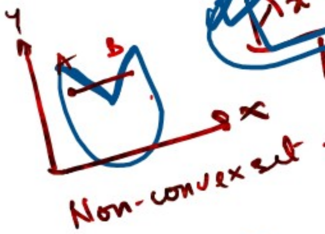
① Convex Sets:

$\lambda x + (1-\lambda) y$

pt A $\Rightarrow (x_1, y_1)$

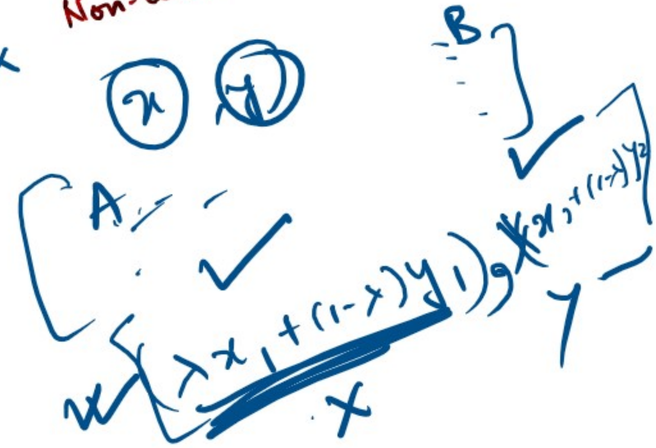
pt B $\Rightarrow (x_2, y_2)$





$\boxed{\lambda x + (1-\lambda) y}$

↳ convex set.



\# A convex set is a point set.

point sets are sets whose elements are point

\# Then a point set is given by a set of points in the two-dimensional plane. If the set contains 'm' points $A_1, A_2, A_3, \ldots, A_m$, the the point set is given by $X = \{A_1, A_2, \ldots, A_m\}$, where $A_1, A_2 \ldots A_m$ are points in 2-D plane.

Suppose we consider any two points A_1 and A_2 from the point set X. The combination of the line segment joining these two points is called a convex combination of A_1 and A_2. Mathematically, this can be written as

...tion ... $(1-\lambda) A_2$

A_1 and A_2

Convex combination can be wri

$$A = \lambda A_1 + (1-\lambda) A_2 \quad \text{where } \boxed{0 \le \lambda \le 1}$$

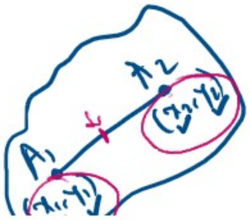
All convex combinations of the finite set of points are point set.

$\therefore$ Convex set are point set.

$\therefore$ A point set is said to be a convex set if the convex combination of any two points of the set is within the set. That is the line segment joining two points of the set should lie within the set or boundary, then it is called a convex set.

Ex: Prove that the set $X = \{(x,y) \mid x+2y \le 5\}$ is a convex set.

Soln: Let us take two points $A_1 : (x_1, y_1)$ and $A_2 (x_2, y_2)$ of the the set X.



$\therefore$ $x_1 + 2y_1 \le 5$ — (1)

$x_2 + 2y_2 \le 5$ — (2)

... A_1 and A_2

$x_2 + 2y_2 \leq 5$

Now the convex combination of A_1 and A_2 is given by $\left[\lambda x_1 + (1-\lambda)x_2 \,,\, \lambda y_1 + (1-\lambda)y_2\right]$

where $0 \leq \lambda \leq 1$.

Now, $\left[\lambda x_1 + (1-\lambda)x_2\right] + 2\left[\lambda y_1 + (1-\lambda)y_2\right]$

$= \lambda x_1 + \underline{(1-\lambda)x_2} + 2\lambda y_1 + \underline{2(1-\lambda)y_2}$

$= \underbrace{\lambda}_{\leq 1}(\underbrace{x_1 + 2y_1}_{\leq 5}) + \underbrace{(1-\lambda)}_{\leq 1}(\underbrace{x_2 + 2y_2}_{\leq 5})$

if $\lambda = 0 \Rightarrow x_2 + 2y_2 \leq 5$ (from (2))

$\lambda = 1 \Rightarrow x_1 + 2y_1 \leq 5$ (from (1))

ie, $\lambda 5 + (1-\lambda) 5$

if $0 \leq \lambda \leq 1$ then $\boxed{\lambda 5 + (1-\lambda) 5 \leq 5}$

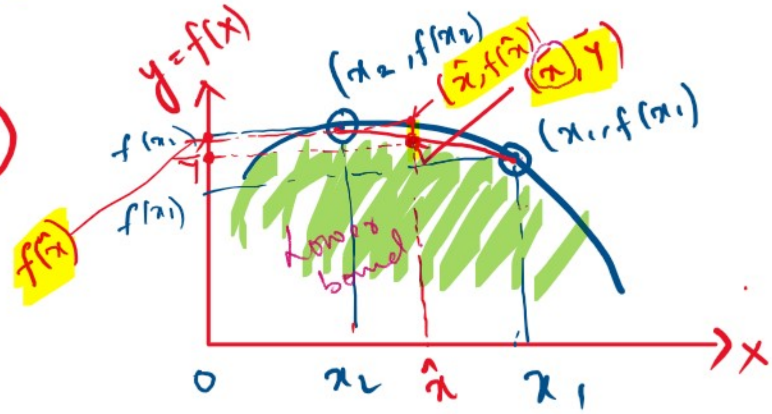
This means that the convex combination of $A_1(x_1, y_1)$ and $A_2(x_2, y_2)$ is also a point of set X.

Hence it is a Convex Set.

... Concave functions:

Convex functions and Concave functions:

(1)



Concave function

↳ change in slope.

Suppose there is an interval (a,b) in $x_1, x_2 \in I$

Now the convex combination of x_1 and x_2 is given by

$$\boxed{\hat{x} = \lambda x_1 + (1-\lambda)x_2}$$

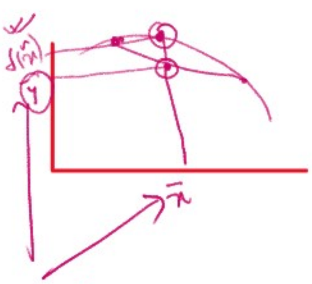
for any $\lambda \in [0,1]$

by cross-sectional formula $\boxed{\bar{x} = \frac{\lambda x_1 + (1-\lambda)x_2}{\lambda + (1-\lambda)}}$

$$\bar{x} = \lambda x_1 + (1-\lambda)x_2$$

and $\bar{y} = \lambda y_1 + (1-\lambda) y_2$

$\bar{y} = \lambda f(x_1) + (1-\lambda) f(x_2)$



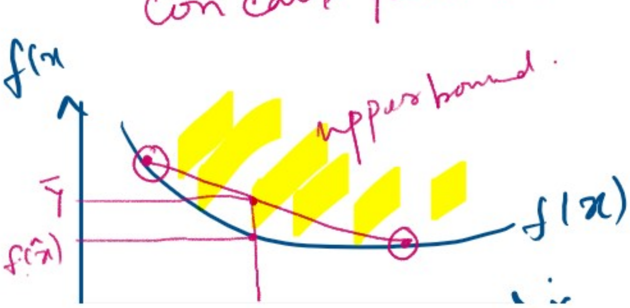
∴ The cordinates are $\left[\underbrace{\{\lambda x_1 + (1-\lambda)x_2\}}_{\hat{x} \text{ or } \bar{x}}, \underbrace{\{\lambda f(x_1) + (1-\lambda) f(x_2)\}}_{\bar{y}}\right]$

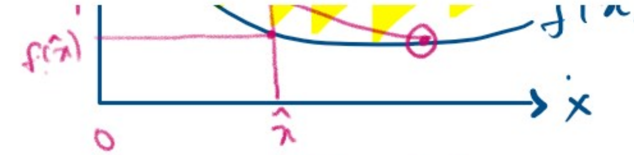
From diagram $f(\hat{x}) > \bar{y}$

i.e. $f\left(\lambda x_1 + (1-\lambda)x_2\right) > \lambda f(x_1) + (1-\lambda) f(x_2)$

↳ This is called strictly concave function.

(2) Convex function:





Strictly convex fn is defined by

$$f(\hat{x}) < \bar{y}$$

or, $$\boxed{f(\lambda x_1 + (1-\lambda)x_2) < \lambda f(x_1) + (1-\lambda) f(x_2)}$$