REAL ANALYSIS SESSION 9/5/2024

09 May 2024 19:38

Limit Points

** Tips: manipulate the given interval to find matching and the given inequality

$$\left\{ \frac{2}{x+1} : x \in (-1,1) \right\} \text{ in } \mathbb{R}$$

$S = \frac{2}{x+1}$ $\quad 0 < x+1 < 2$

$-1 < x < 1$

$$\frac{1}{2} < \frac{1}{x+1} < \infty$$

$$1 < \frac{2}{x+1} < \infty$$

$S \to (1, \infty)$

Limit pts are $[1, \infty)$

(2) $S = \left\{ \frac{1}{3^n} + \frac{1}{7^m} \right\}$ $n, m \in \mathbb{N}$

S is ~~closed~~/open/compact / 0 is a LP of S.

Ans $S = \frac{1}{3^n} + \frac{1}{7^m}$ $\quad n, m \in \mathbb{N}$

$0 \notin S \to S$ is not closed

$\frac{1}{3} + \frac{1}{7} = \frac{10}{21} \in S$

(... is an Empty set)

$\frac{1}{3} + \frac{1}{7} = \frac{10}{21} \in S$

But $\frac{10}{21} \notin S^0$ (which is an Empty set)

[Claim: $S^0 = \phi$ $\quad S \subseteq \mathbb{Q}$ (Rationals)

$\Rightarrow S^0 \subseteq \mathbb{Q}^0$

$S^0 = \phi$]

Connectedness

$S\ (\subseteq \mathbb{R})$ is connected iff either S is an interval or singleton; but here S is neither.

Logic

$\frac{16}{63}, \frac{10}{21} \in S$

$\frac{16}{23} < \frac{12}{35} < \frac{10}{21}$ but $\frac{12}{35} \notin S$

[for $n=2$, $m=1$, $\frac{1}{3^2} + \frac{1}{7} = \frac{16}{63}$

$n=1$, $m=1$ $\quad \frac{1}{3} + \frac{1}{7} = \frac{10}{21}$]

So, S is not an interval $\longrightarrow$ NOT Connected

Point Based

PMF

PMF

$$\lim_{\substack{n\to\infty \\ m\to\infty}} \left(\frac{1}{3^n} + \frac{1}{7^m}\right) = 0$$

0 is the limit of S

B ✓ & ✓

Better sheet

Marble

Rasi Induction

Academic Pahlu

Solved

NBHM 2019

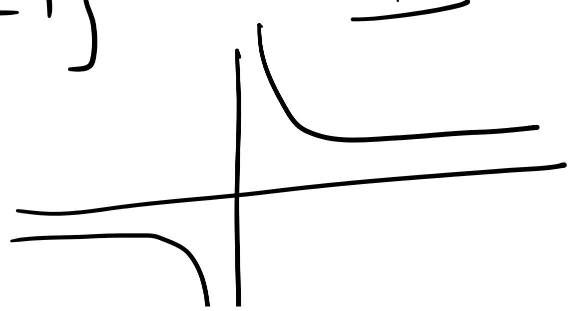
Which are Compact of $\mathbb{R}^2$

a) $\{(x,y) : xy = 1\}$

$\{(x,y) : x^{2/3} + y^{2/3} = 1\}$

$x^2 + y^2 < 1\}$

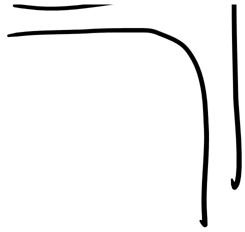
R/A



let $\{(x,y) \mid xy = 1\}$

Let, $\{(x,y) \mid x \cdot y = 1\}$

$y = \frac{1}{x} = x^{-1}$

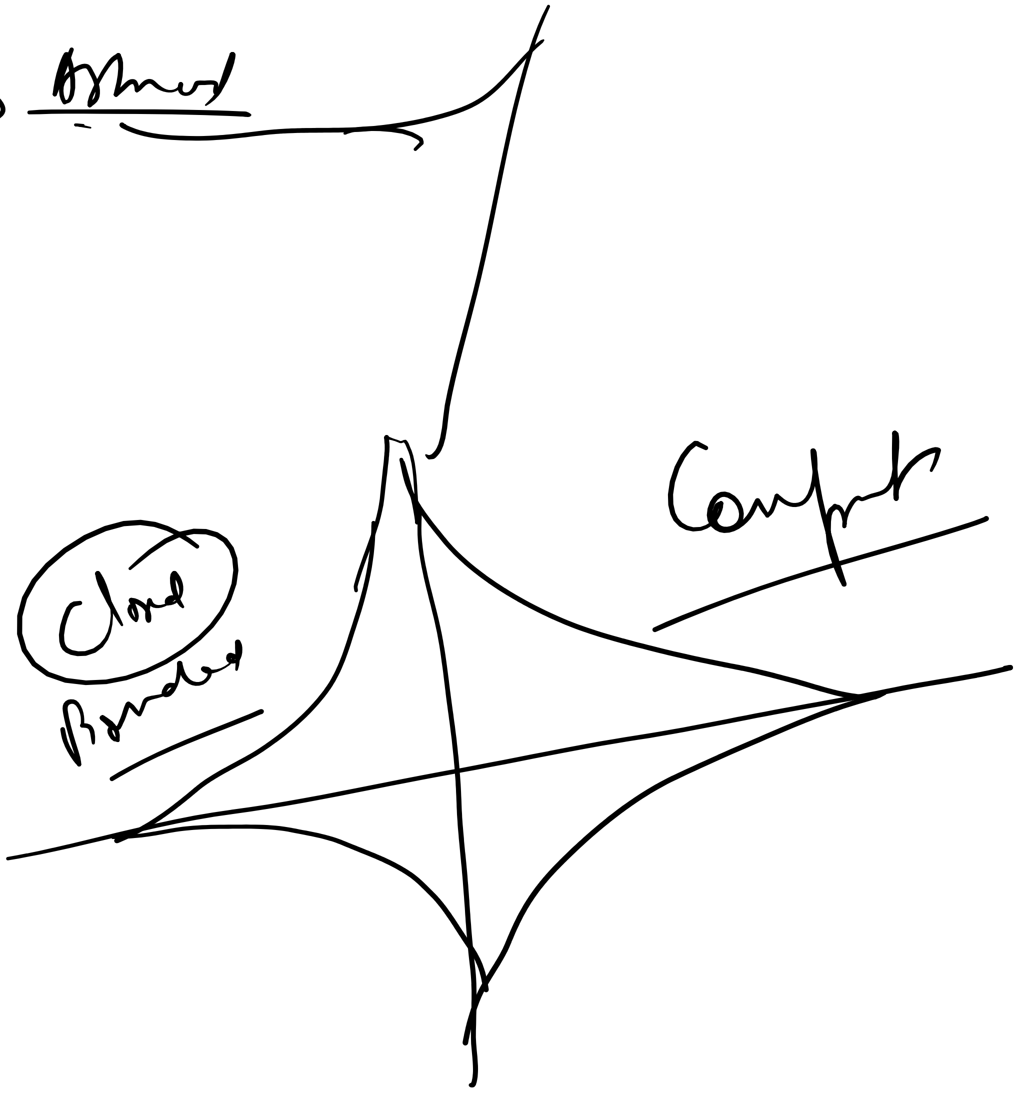


Unbounded

NOT Compact

$x^{2/3}+y^{2/3} = 1$

→ Astroid



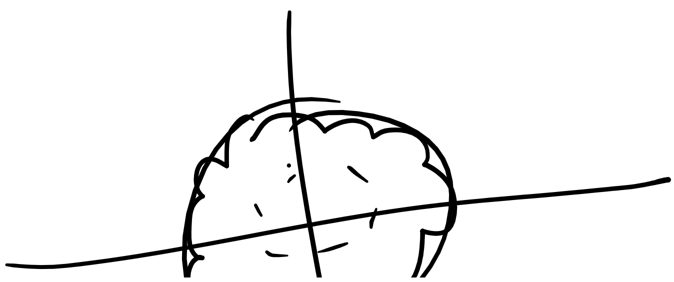
Closed

Bounded

Compact

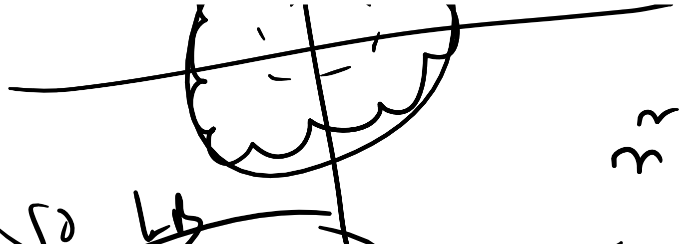
$x^2+y^2 < 1$

Open disc



NO

NB → No LB, not complete

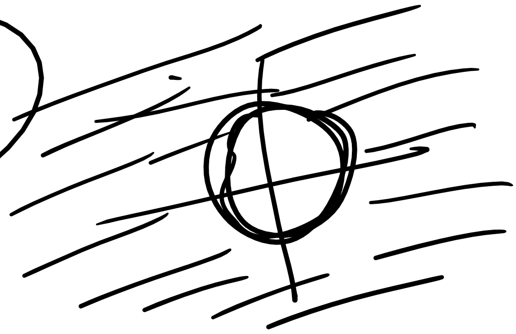


x^n fxy^2 0.5

0.4

0.04

0.0004



Sup Inf

$$A = \left\{ t \sin\left(\frac{1}{t}\right) \;\middle|\; t \in \left(0, \frac{2}{\pi}\right) \right\}$$

$\frac{2}{\pi} < \frac{2}{\pi} + \frac{1}{4\pi}$ $\forall n \geq 1$

Sup(A) =

Sup. $u_n = \frac{4+n}{n} = \frac{4}{n} + 1$

5, 1 inf

sup

Ann Dragon Bros

Inf(A) = 0 > $-\frac{2}{3\pi} - \frac{1}{4\pi}$ $\forall n \geq 1$

(K) $E = \left\{ \frac{n}{n+1}, n \in N \right\}$

$F = \left\{ \frac{1}{1+x} : 0 \leq x < 1 \right\}$

$$F = \left\{ \frac{1}{1-x} : 0 \le x \ldots \right\}$$

$$E = \frac{n}{n+1} = \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5} \ldots \quad \to E \text{ Bounded}$$

$$\frac{1}{2} \le x < 1 \quad \forall x \in E$$

$$1 \notin E \quad \left(\because \text{if } 1 \in E \Rightarrow \frac{n}{n+1} = 1, \; n = n+1, \; 1 = 0 \right)$$

But as E contain ∞ many values in neighborhood of 1, so E is not closed.

$$F = \frac{1}{1-x} \qquad 0 \le x < 1$$

$$-1 < -x \le 0$$

$$0 < 1-x \le 1$$

$$1-x \le 1 \Rightarrow \frac{1}{1-x} \ge 1 \qquad F \text{ is closed} \ldots$$

Limit Points

$$Y = \left\{ \frac{x}{1+|x|} \right\} \; x \in R \qquad \text{Set All limit pts of } Y$$

$$= \frac{|x|}{\ldots |x|}$$

As, $\left|\frac{x}{1+|x|}\right| = \frac{|x|}{|1+|x||} = \frac{|x|}{1+|x|}$ $\left[\because \frac{1}{1+|x|} > 0\right]$

$= 1 - \frac{1}{1+|x|} < 1$

So $\left|\frac{x}{1+|x|}\right| < 1 \ \forall x \in \mathbb{R}$

$-1 < \frac{x}{1+|x|} < 1$

$Y = (-1, 1)$ ← Set of last prob

ISI MSQE 2014 — DSE, JNU → CUET

$\left\{\frac{x^2}{1+x^2} : x \in \mathbb{R}\right\}$

$x \in \mathbb{R}$, $x^2 \geq 0$ $1+x^2 \geq 0$

$\frac{x^2}{1+x^2} \geq 0 \ \forall x \in \mathbb{R}$

$\frac{x^2}{1+x^2} < 1$ and $\frac{x^2}{1+x^2} \neq 1$

$S = [0, 1)$ ← closed [, not closed)

S is Bounded But Not closed..

But not Compact..

Connected But not [illegible]