Q. The maximum value of $a(1-a)b(1-b)c(1-c)$ where a, b, c are positive fractional values is!

(a) 1 (b) $\frac{1}{8}$ (c) $\frac{1}{27}$ ~~(d)~~. $\frac{1}{64}$.

$f(a,b,c) = a(1-a)\cdot b(1-b)\,c(1-c)$.

6 terms: $a, (1-a), b, (1-b), c(1-c)$

$$AM = \frac{\cancel{a} + (1-\cancel{a}) + \cancel{b} + (1-\cancel{b}) + \cancel{c} + (1-\cancel{c})}{6} = \frac{3}{6} = \frac{1}{2}.$$

$$GM = \left\{a(1-a)\,b(1-b)\,c(1-c)\right\}^{\frac{1}{6}}.$$

$\therefore\ AM \geq GM \Rightarrow \frac{1}{2} \geq \left\{a(1-a)\,b(1-b)\,c(1-c)\right\}^{\frac{1}{6}}$

$$a(1-a)\,b(1-b)\,c(1-c) \leq \left(\frac{1}{2}\right)^6 = \frac{1}{64}.$$

Q. $\sum_{i=1}^{4} x_i = 2$ and $x_i \geq 0\ \forall i$. Then $P = (x_1+x_2)\cdot(x_3+x_4)$ is bounded b/w: ~~(a)~~ 0 and 1 (c) 2 and 3

(b) 1 and 2 (d) 3 and 4.

$a = (x_1 + x_2)$

$b = (x_3 + x_4)$

$AM \geq GM$.

$$\frac{2}{2} \geq \left\{(x_1+x_2)\cdot(x_3+x_4)\right\}^{\frac{1}{2}}.$$

$$(x_1+x_2)(x_3+x_4) \leq 1.$$

Q. Let $a^2+b^2+c^2 = 1$. Then $ab+bc+ca = ?$

(a) -0.75 (b) $\in \left(-\frac{1}{2}, 1\right)$ (c) $\in \left(-\frac{1}{2}, \frac{1}{2}\right)$ ~~(d)~~ None.

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$(a+b+c)^2 \geq 0.$

$a^2+b^2+c^2+2(ab+bc+ca) \geq 0.$

$1 + 2(ab+bc+ca) \geq 0.$

$ab+bc+ca \geq -\frac{1}{2}$ $\therefore$ $ab+bc+ca \in \left[-\frac{1}{2}, \infty\right)$

Q. The least strictly positive value of $a^3+b^3+c^3-3abc$, where a, b, c are strictly positive integers is:

(a) 2 (b) 3 ~~(c)~~ 4 (d) 8.

Put $a=1$, $b=1$, $c=2$.

$1^3+1^3+2^3-3\times 2 = 1+1+8-6 = 4.$

$a=b=c=1$

$a^3+b^3+c^3-3abc = 0.$

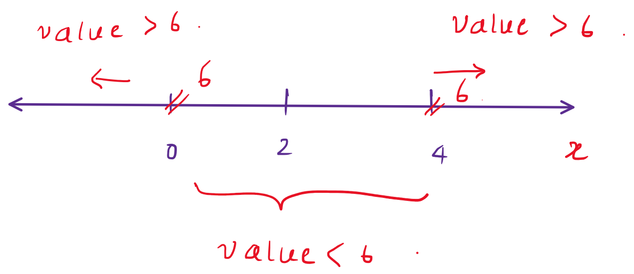
Q. Solve for x: $|x-1|+|x-2|+|x-3| \geq 6.$

$x=2$:

$x=4$: $\Rightarrow$ value $= 6$.

$x=5$: $\Rightarrow$ value > 6.

$x=0$: $\Rightarrow$ value $= 6$



$$|x-1| = \begin{cases} (x-1), & x \geq 1 \\ -(x-1), & x < 1. \end{cases}$$

$|x-2| = \{$

$|x-3| = \{$

$|x-1| + |x-2| + |x-3|$

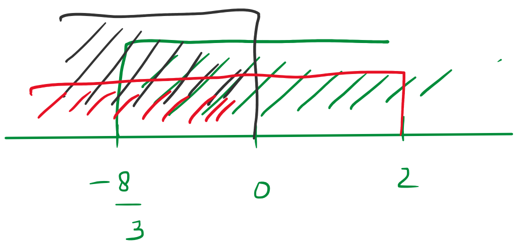
For, $x < 1$: $-(x-1) - (x-2) - (x-3)$.

$1 \leq x < 2$: $(x-1) - (x-2) - (x-3)$.

$2 \leq x < 3$: $(x-1) + (x-2) - (x-3)$

$x \geq 3$: $(x-1) + (x-2) + (x-3)$

Q. Solve for x : $\underbrace{|2x+3|}_{x = -3/2} + \underbrace{|x+1|}_{x = -1.} \leq 4$.



$\therefore$ $|2x+3| + |x+1|$

For $x < -\frac{3}{2}$: $-(2x+3) - (x+1) \leq 4 \Rightarrow -3x - 4 \leq 4 \Rightarrow x \geq -\frac{8}{3}$.

For $-\frac{3}{2} \leq x < -1$: $(2x+3) - (x+1) \leq 4 \Rightarrow x \leq 2$.

For $x \geq -1$: $(2x+3) + (x+1) \leq 4 \Rightarrow x \leq 0$.

$\therefore$ $x \in \left[-\frac{8}{3}, 0\right]$.

Q. The remainder when $1! + 2! + \cdots + 95!$ is divided by 15 is : (a) 14 (b) 3 (c) 1 (d) None.

$$\frac{1! + 2! + 3! + 4!}{15} = \frac{1 + 2 + 6 + 24}{15} = \frac{33}{15} \Rightarrow \text{Rem} = 3.$$

$$\left(\frac{1! + 2! + 3! + 4!}{15}\right) + \left(\frac{\overset{3\times5}{5!} + \overset{3\times5}{6!} + \cdots + \overset{3\times5}{95!}}{15}\right)$$

HW.

Q. For every positive integere of the form: $(n^3 - n)(n^2 - 4)$ $n = 3, 4, \ldots$ is:

(a) Divisible by 12.

(b) Divisible by 24

(c) Divisible by 120.

(d) Divisible by 720