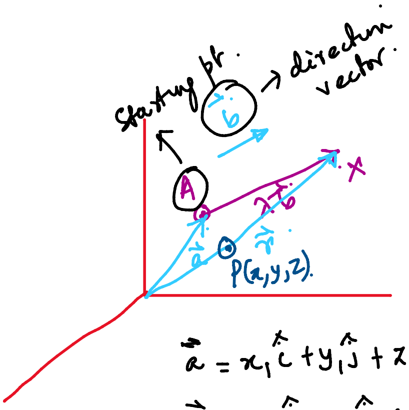
30 December 2023 10:03

VECTOR EQUATIONS.

St line | Plane.



Any vector parallel to a given vector $\vec{b}$ can be represented by $\lambda\vec{b}$ where λ is a scalar constant.

$\vec{r} = \vec{a} + \vec{AX} = \vec{a} + \lambda\vec{b}$

$\boxed{\vec{r} = \vec{a} + \lambda\vec{b}}$ → vector equation of a st line.

$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$P(x,y,z) = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{r} = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda(x_2\hat{i} + y_2\hat{j} + z_2\hat{k})$

$= \boxed{(x_1+\lambda x_2)}\hat{i} + \boxed{(y_1+\lambda y_2)}\hat{j} + \boxed{(z_1+\lambda z_2)}\hat{k}$

$x = x_1 + \lambda x_2 \qquad y = y_1 + \lambda y_2 \qquad z = z_1 + \lambda z_2$

$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$

$\dfrac{x-x_1}{x_2} = \lambda \qquad \dfrac{y-y_1}{y_2} = \lambda \qquad \dfrac{z-z_1}{z_2} = \lambda$

Coordinates of the starting pt. (A)

$\boxed{\dfrac{x-x_1}{x_2} = \dfrac{y-y_1}{y_2} = \dfrac{z-z_1}{z_2}}$ → equ of a line in the cartesian form.

Coordinates of the direction vector. $(\vec{b})$

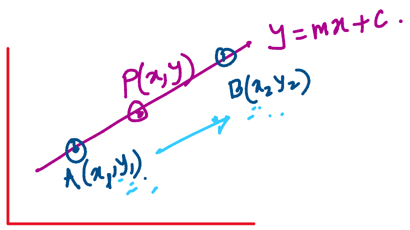
vector form

Starting pt. direction vector.

$\vec{r} = \vec{a} + \lambda\vec{b}$ ⟷ $\boxed{\dfrac{x-x_1}{x_2} = \dfrac{y-y_1}{y_2} = \dfrac{z-z_1}{z_2}} = \lambda$ Cartesian form

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

and $\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

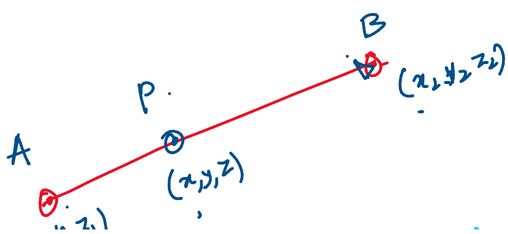


Slope of AP $= \dfrac{y-y_1}{x-x_1} = m.$

Slope of AB $= \dfrac{y_2-y_1}{x_2-x_1} = m.$

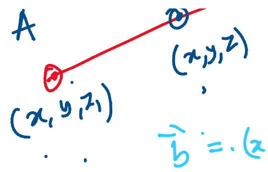
$\dfrac{y-y_1}{x-x_1} = \dfrac{y_2-y_1}{x_2-x_1}$

$\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$



Slope of AP = Slope of AB.

$\dfrac{x-x_1}{\ldots} = \dfrac{y-y_1}{\ldots} = \dfrac{z-z_1}{z_2-z_1}$



$$\vec{b} = (x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}$$

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

If the lines $\vec{r}_1 = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{r}_2 = p\hat{i} - 2p\hat{j} + 3\hat{k}$ are perpendicular to each other find p.

$\vec{r}_1 \cdot \vec{r}_2 = 0$.

Condition for orthogonality (perpendicular) is that the dot product $= 0$

$$2p - 6p + 3 = 0$$
$$-4p + 3 = 0.$$
$$\boxed{p = 3/4}$$

Find the equation of a line passing through $\boxed{(1,2,3)}$ and parallel to $\boxed{\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}}$ in vector form and in cartesian form.

→ starting pt.

direction vector

Vector form

$$\vec{r} = \langle \text{starting pt} \rangle + \lambda \langle \text{direction vector} \rangle$$
$$= (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$$

Catesian form

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{2}.$$

Find the eqn of the line passing through $P(1,2,3)$ and $Q(2,-1,-3)$ in vector and cartesian forms.

x_1, y_1, z_1 $\quad x_2\ y_2\ z_2$.

starting pt $= P$ $\quad$ direction vector $= Q - P$.

$$\vec{r} = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda[(x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}]$$

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - 3\hat{j} - 6\hat{k})$$

$$\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z-3}{-6}$$

The equation of a line is $\boxed{x+2 = \frac{3-y}{2} = \frac{z-1}{-2}}$. find the equation in vector form.

Starting pt $\cancel{(2,-3,1)}$ $(-2,3,1)$ $\quad$ direction vector $(1, \cancel{2}, -2)$ $(1,-2,-2)$

$$\frac{x-x_1}{x_2} = \frac{y-y_1}{y_2} = \frac{z-z_1}{z_2}.$$

$$\frac{x-(-2)}{1} = \frac{y-3}{-2} = \frac{z-1}{-2}.$$

starting pt.

$(1,2,3)$ is $\vec{r} = 4\hat{i} - \hat{j} - 2\hat{k}$

The equation of a line passing through $(1,2,3)$ is $\vec{r} = 4\hat{i} - \hat{j} - 2\hat{k}$. find the equation in the cartesian form.

Starting pt.

$$\vec{r} = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda(x_2\hat{i} + y_2\hat{j} + z_2\hat{k})$$

$x_1 + \lambda x_2 = 4.$

$1 + \lambda x_2 = 4$

$\boxed{\lambda x_2 = 3}$

$\boxed{x_2 = \frac{3}{\lambda}}$

$y_1 + \lambda y_2 = -1$

$2 + \lambda y_2 = -1$

$\boxed{\lambda y_2 = -3}$

$\boxed{y_2 = -\frac{3}{\lambda}}$

$z_1 + \lambda z_2 = -2.$

$3 + \lambda z_2 = -2.$

$\boxed{\lambda z_2 = -5}$

$\boxed{z_2 = -\frac{5}{\lambda}}$

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\left(\frac{3}{\lambda}\hat{i} - \frac{3}{\lambda}\hat{j} - \frac{5}{\lambda}\hat{k}\right).$$

direction vector.

$$\frac{x-1}{\frac{3}{\lambda}} = \frac{y-2}{\frac{-3}{\lambda}} = \frac{z-3}{\frac{-5}{\lambda}}.$$

$$\frac{\cancel{\lambda}(x-1)}{3} = \frac{\cancel{\lambda}(y-2)}{-3} = \frac{\cancel{\lambda}(z-3)}{-5}$$

$$\frac{x-1}{3} = \frac{y-2}{-3} = \frac{z-3}{-5} \rightarrow \boxed{\frac{x-1}{3} = \frac{2-y}{3} = \frac{3-z}{5}}$$