Q. If $H = f(\underbrace{y-z}_{p}, \underbrace{z-x}_{q}, \underbrace{x-y}_{r})$. $\dfrac{\partial H}{\partial x} + \dfrac{\partial H}{\partial y} + \dfrac{\partial H}{\partial z} =$ (a) 0 (b) 1 (c) -1 (d) None.

$H = f(p, q, r)$ $p = y - z$, $q = z - x$, $r = x - y$.

$$\frac{\partial H}{\partial x} = \frac{\partial H}{\partial q}\underbrace{\left(\frac{\partial q}{\partial x}\right)}_{(-1)} + \frac{\partial H}{\partial r}\underbrace{\left(\frac{\partial r}{\partial x}\right)}_{1} = -\frac{\partial H}{\partial q} + \frac{\partial H}{\partial r}.$$

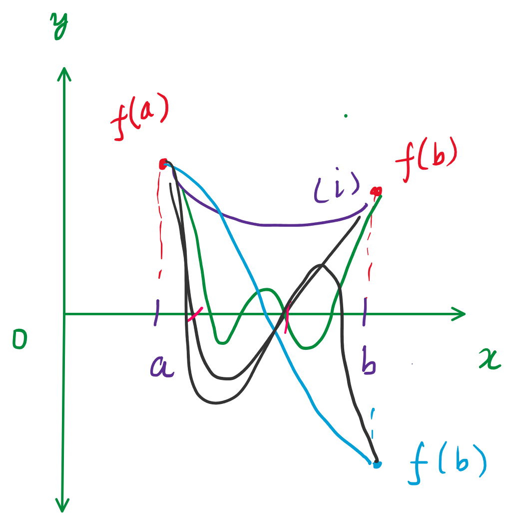
$$\frac{\partial H}{\partial y} = \frac{\partial H}{\partial p} - \frac{\partial H}{\partial r}$$

$$\frac{\partial H}{\partial z} = \frac{\partial H}{\partial q} - \frac{\partial H}{\partial p}$$

$$\therefore \frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0. \text{ (a)}$$

Quadratic and Cubic Equations.

Let $f(x)$ be a polynomial of any degree.
[continuous & differentiable].



Case I: $f(a)$ and $f(b)$ have the same sign.

$\Rightarrow$ Either there are no roots / even no. of roots.

Case II: $f(a)$ and $f(b)$ have opposite signs.

$\Rightarrow$ Odd no. of roots in the given interval.

Quadratic Equation :-

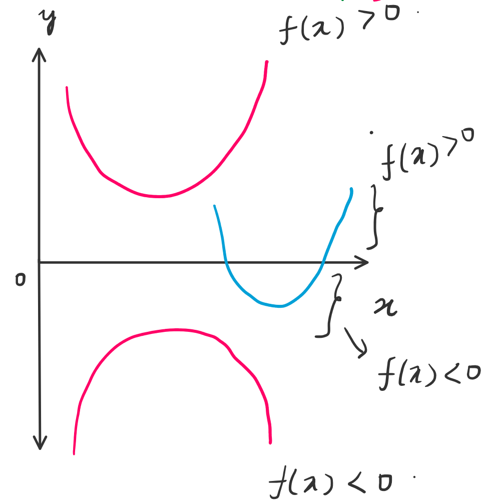
$$f(x) = ax^2 + bx + c, \quad a \neq 0.$$

2 Roots of $f(x)$ are $\alpha, \beta \Rightarrow \alpha + \beta = -\frac{b}{a}, \ \alpha\beta = \frac{c}{a}.$

$f(x)$ can also be represented as : $f(x) = a\left[x^2 - (\alpha+\beta)x + \alpha\beta\right]$

Case I: Roots are complex :

Roots are : $\alpha + i\beta, \ \alpha - i\beta$.



$$f(x) = a\left[x^2 - (\alpha+\beta)x + \alpha\beta\right].$$

$$= a\left[(x-\alpha)(x-\beta)\right].$$

$$= a\left[\{x - (\alpha + i\beta)\}\{x - (\alpha - i\beta)\}\right].$$

$$= a\left[\{(x-\alpha) - i\beta\}\{(x-\alpha) + i\beta\}\right]$$

$$= a\left[(x-\alpha)^2 - (i\beta)^2\right]$$

$$f(x) = a\left[\underbrace{(x-\alpha)^2 + \beta^2}_{>0}\right].$$

$f(x) > 0 \Rightarrow a > 0$.
$f(x) < 0 \Rightarrow a < 0$.
} Depends on sign of 'a'.

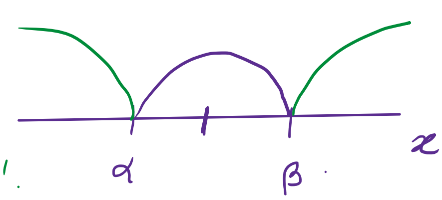
Case II: Roots are Real & Equal : Root $= \alpha$.

$$f(x) = a(x-\alpha)(x-\alpha) = a(x-\alpha)^2 \ \text{--- sign of 'a'.}$$

Case III: Roots are Real & Unequal : Roots $= \alpha, \beta \ [\alpha < \beta]$.

$$f(x) = a(x-\alpha)(x-\beta).$$

$x > \beta$: $\quad \underbrace{(x-\alpha)}_{>0} \ \underbrace{(x-\beta)}_{>0}$



sign of $f(x)$ depends upon sign of 'a'.

$x < \alpha$: $f(x) = a\,\underbrace{(x-\alpha)}\,\underbrace{(x-\beta)}$

$\underbrace{\underbrace{\quad}_{<0}\qquad\underbrace{\quad}_{<0}}_{>0}$.

Sign of $f(x)$ depends upon sign of 'a'.

$\alpha < x < \beta$: $f(x) = a\,\underbrace{\underbrace{(x-\alpha)}_{>0}\,\underbrace{(x-\beta)}_{<0}}_{<0}$.

If $a > 0 \Rightarrow f(x) < 0$.

If $a < 0 \Rightarrow f(x) > 0$.

Conclusion: Sign of quadratic is same as the sign of 'a' except when roots are real & unequal and x has a value lying b/w them.

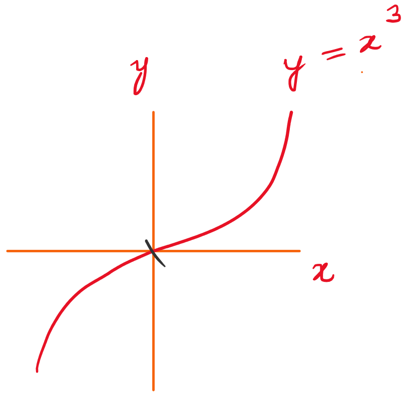
& Nature of roots of any quad ax^2+bx+c can be known by checking $D = b^2 - 4ac$.

$D > 0 \Rightarrow$ Roots are real & unequal.

$D = 0 \Rightarrow$ Roots are real & equal.

$D < 0 \Rightarrow$ Roots are complex.

Cubic Equations



$f(x) = x^3 + ax^2 + bx + c$.

$f'(x) = 3x^2 + 2ax + b$.

↳ sign of quad determine increasing/decreasing nature of $f(x)$

This will be determined by nature of roots of $f'(x)$

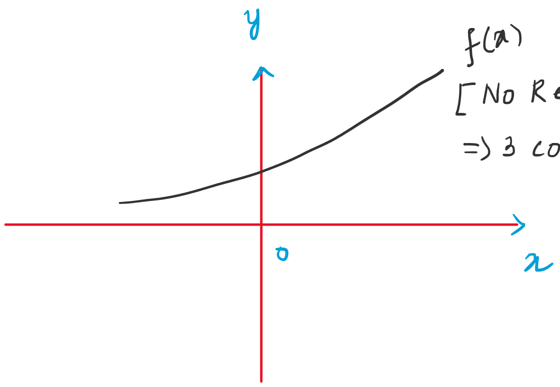
$D_{f'(x)} = (2a)^2 - 4(3)(b) = 4a^2 - 12b = 4(a^2 - 3b)$

Case I: If $D_{f'(x)} < 0 \Rightarrow$ sign of $f'(x)$ depends on coeff of x^2

Case I: If $D_{f'(x)} < 0 \Rightarrow$ sign of $f'(x)$ depends on coeff of x^2

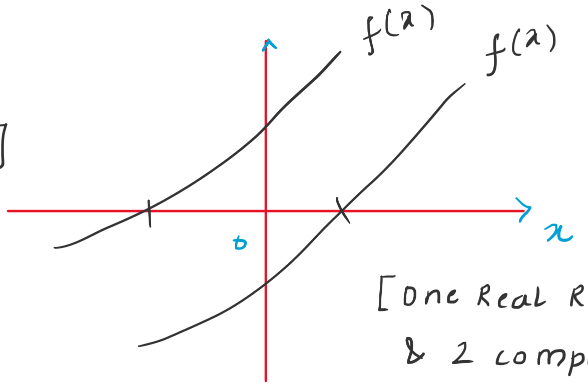
Here $f'(x) = 3x^2 + 2ax + b \Rightarrow f'(x) > 0$ [$f(x)$ is increasing]

> 0



[No Real Roots $\Rightarrow$ 3 complex Roots] ✗

(i) [Not possible]



[One Real Roots & 2 complex Roots]

(ii)

HW Case II: $D_{f'(x)} > 0$.