Recall: Test $H_0: \mu = \mu_0$ vs $H_1: \mu = \mu_1$ [Consider a Normal population]

Consider a random sample: $X_1, X_2, \dots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$.

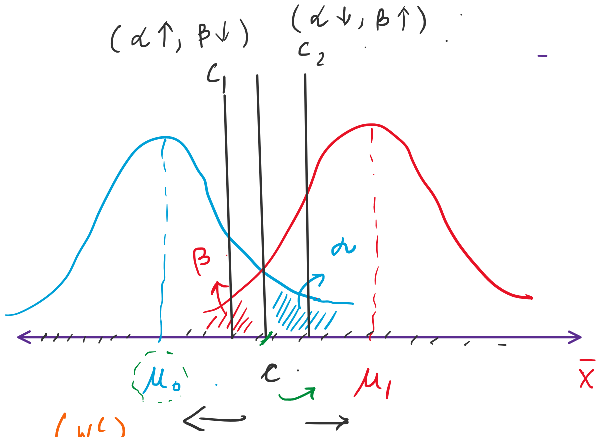
$\therefore$ We know $E(\bar{X}) = \mu$.

$$Var(\bar{X}) = \frac{\sigma^2}{n}$$

and $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

Under H_0: $\bar{X} \sim N\left(\mu_0, \frac{\sigma^2}{n}\right)$

Under H_1: $\bar{X} \sim N\left(\mu_1, \frac{\sigma^2}{n}\right)$



To test the hypothesis we need fix a critical value:-

If $\bar{X} < c \Rightarrow H_0$ is true.

If $\bar{X} > c \Rightarrow H_1$ is true.

2 Error situations: (i) Type I Error $(\alpha) = P[\bar{X} > c \mid H_0] = P[\bar{X} \in W \mid H_0]$

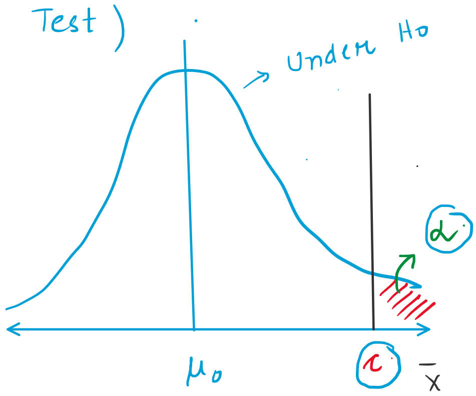
(ii) Type II Error $(\beta) = P[\bar{X} < c \mid H_1] = P[\bar{X} \in W^c \mid H_1]$

Note: Although α, β depends choice of critical point 'c', there does not exist any 'c' s.t both α, β are simultaneously minimized.

Hence fix α to a pre-determined level & then try to minimize Type II Error (or max. the Power of the Test).

This pre-determined level of α is known as Level of significance of test.

[usually $\alpha = 1\% (= 0.01) / 5\% (= 0.05)$].



Note: Result:

Eg: Given the sample we observe $\bar{x} > c$.

We will reject H_0 [at $\alpha\%$ L.O.S.]

Eg (1) Suppose r.s $X_1, X_2, \dots, X_n \overset{iid}{\sim} N(\mu, 1)$ popln.

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To test: $H_0: \mu = \mu_0$ vs $H_1: \mu = \mu_1 \ (> \mu_0)$

(i) First calculate $\bar{X}$ from the sample, $\boxed{\bar{X} \sim N\left(\mu, \frac{1}{n}\right)}$

(ii) To fix α $(=0.05)$ and accordingly getting c, then

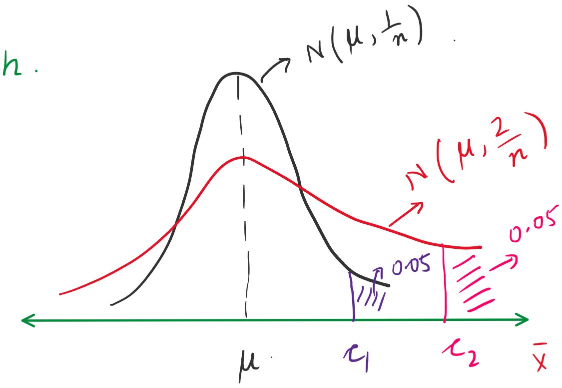
(iii) Compare $\bar{X}$, c to conclude.

Eg (2) Suppose r.s $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, 2)$ popln.

To test $H_0: \mu = \mu_0$ vs $H_1: \mu = \mu_1 \ (> \mu_0)$.

Fix $\alpha = 0.05$ & 'n' is same for both.

Here $\bar{X} \sim N\left(\mu, \frac{2}{n}\right)$



Note: $X \sim N(\mu, \sigma^2)$.

$$\tau = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

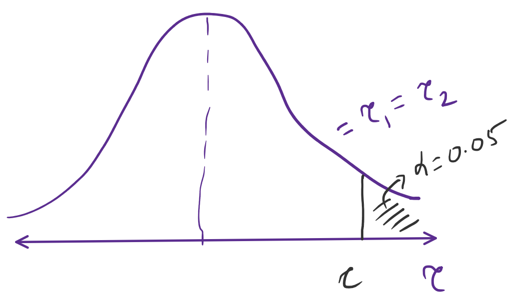
Eg (1) $\bar{X} \sim N\left(\mu, \frac{1}{n}\right) \Rightarrow \tau_1 = \dfrac{\bar{X} - \mu}{\sqrt{1/n}} \sim N(0, 1)$

Eg (2) $\bar{X} \sim N\left(\mu, \frac{2}{n}\right) \Rightarrow \tau_2 = \dfrac{\bar{X} - \mu}{\sqrt{2/n}} \sim N(0, 1)$

critical pts to be used is the same.

Note: These transformations are known as Test statistic.

The test-statistic will always be constructed in a manner such that they follow the standard distributions: τ, χ^2, t, F, so that the known critical values are used.



$\therefore$ Procedure for Testing of Hypothesis:

(i) Consider the r.s and construct the test-statistic.

(ii) Fix α, to get the critical point and conclude.

Q. If $x \geq 1$ is the critical region for testing $H_0: \theta = 2$ vs $H_1: \theta = 1$ on the basis of a single obs from the pop ln $f(x) = \theta e^{-\theta x}, \ 0 \leq x < \infty$. Find the type I & type II errors.

$W: x \geq 1 \qquad W^c: 0 \leq x < 1$.

To test: $H_0: \theta = 2$ vs $H_1: \theta = 1$.

R.S of size $X_1 \sim f(x)$.

$$\alpha = P[x \in W \mid H_0] = P[x \geq 1 \mid \theta = 2]$$

$$= \int_1^{\infty} 2e^{-2x}\,dx = \frac{1}{e^2}$$

$$\beta = P[x \in W^c \mid H_1] = P[0 \leq x < 1 \mid \theta = 1]$$

$$= \int_0^1 e^{-x}\,dx = \left(1 - \frac{1}{e}\right)$$

Q. Let 'p' denote the prob of getting a head in a single toss of a coin. We want to test $H_0: p = 1/2$ vs $H_1: p = 3/4$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find Type I Error, Power of the Test.

X: R.V indicates no. of heads obtained in 5 tosses of a coin

$\therefore X \sim Bin(5, p) \Rightarrow f(x) = {}^5C_x \, p^x (1-p)^{5-x}, \ 0 \leq x \leq 5$.

$W = \{4, 5\} \qquad W^c = \{0, 1, 2, 3\}$.

HW $\alpha = P[X \in W \mid H_0] = P[X \in \{4, 5\} \mid p = 1/2]$

HW $\alpha = P[X \in W / H_0] = P[X \in \{4, 5\} / p = 1/2]$

$= P[X = 4 / p = 1/2] + P[X = 5 / p = 1/2]$

$= {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^1 + {}^5C_5 \left(\frac{1}{2}\right)^5$

$=$

HW $\beta = P[X \in W^c / H_1] = P[X \in \{0, \dots, 3\} / p = 3/4]$

$= 1 - P(X = 4 / p = 3/4) - P(X = 5 / p = 3/4) -$

$=$

Power $= 1 - \beta =$