03 April 2024 09:15 PM

one - sample test

paired - test

two - sample test .

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\hat{\sigma}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

where $\hat{\sigma} = \frac{n_1 \sigma_1^2 + n_2 \sigma_2^2}{n_1+n_2-2}$

$$= \frac{\bar{x}_1 - \bar{x}_2}{\hat{\sigma}\sqrt{\frac{n_2+n_1}{n_1 n_2}}} = \frac{\bar{x}_1 - \bar{x}_2}{\hat{\sigma}}\sqrt{\frac{n_1 n_2}{n_2+n_1}}$$

$\frac{dy}{dt}$

y_t y_{t+1}

subtract

Difference Equations

(1) Case of Homogenious equations

$$y_{t+1} - a y_t = 0$$

Let the trial solution be $y_t = \beta x^t$

$\therefore y_{t+1} = \beta x^{t+1}$

$$\beta x^{t+1} - a\beta x^t = 0$$

$\beta x^t . x^1$

βx^{t+1}

$$\beta x^t (x - a) = 0$$

$\therefore x - a = 0$

$$\boxed{x = a}$$

$\therefore y_t = \beta a^t$

From initial condition at $t=0$

$y_0 = \beta a^0$

$\therefore$ Required solution is $\boxed{y_t = y_0 \cdot a^t}$ (ans) $\beta = y_0$.

Q. Find out the solution for $y_{t+1} - y_t = 4y_t$ when $y_0 = 6$

$$\boxed{y_{t+1} - 5y_t = 0}$$

Let the trial solution be $y_t = \beta x^t$

$\therefore y_{t+1} = \beta x^{t+1}$

$\therefore y_{t+1} - 5y_t = 0$

or, $\beta x^{t+1} - 5\beta x^t = 0$

or, $\beta x^t (x-5) = 0$

$\because \beta x^t = y^t \neq 0 \quad \therefore x - 5 = 0$

$x = 5$

$\therefore y_t = \beta 5^t$

From question at $t = 0 \Rightarrow y_0 = 6$

or $y_0 = \beta 5^0$

or, $6 = \beta$

$\therefore$ Required solution is $\boxed{y_t = 6 \cdot 5^t}$

Q. Find the time path of difference equation $2y_{t+1} - 4y_t = 0$;

$$\boxed{y_{t+1} - 2y_t = 0}$$

Difference Equation in case of Non-Homogeneous fn.

$y_{t+1} - a y_t = b$ ← Non homogeneous.

PI : Let us assume value of y does not change over time i.e $y_{t+1} = y_t = \bar{y}$

$$\therefore \bar{y} - a\bar{y} = b$$

$$\bar{y}(1-a) = b$$

$$\bar{y} = \frac{b}{1-a} \rightarrow \text{PI}$$

↳ equilibrium.

CF : Let us take the homogeneous part of eqn

i.e $\boxed{y_{t+1} - ay_t = 0}$

Let the trial solution be $y_t = \beta x^t$

$$\therefore \beta x^{t+1} - a\beta x^t = 0$$

$$x = a$$

$$\therefore y_t = \beta a^t$$

$\therefore$ The required solution is $y_t = \text{PI} + \text{CF}$

$$= \bar{y} + \beta a^t$$

$$\boxed{y_t = \frac{b}{1-a} + \beta a^t}$$

Initial condition, at $t=0$ $y_0 = \frac{b}{1-a} + \beta$

$$\therefore \beta = \left(y_0 - \frac{b}{1-a}\right)$$

$$\therefore y_t = \frac{b}{1-a} + \left(y_0 - \frac{b}{1-a}\right) a^t \quad \text{(ans)}$$

Q Obtain the solution of the equation $1y_{t+1} - 2y_t = 4$ given that $y_t = 4$ when $t = 0$.

PI Let $y_{t+1} = y_t = \bar{y}$ (PI)

$\therefore \bar{y} - 2\bar{y} = 4$

$-\bar{y} = 4$

$\bar{y} = -4$

CF $y_t = \beta x^t \therefore y_{t+1} = \beta x^{t+1}$

So, $\beta x^{t+1} - 2\beta x^t = 0$

$\beta x^t [x - 2] = 0$

$x - 2 = 0$

$x = 2$

$\therefore y_t = \beta 2^t$

$\therefore$ Required soln is $y_t = -4 + \beta 2^t$

$t=0$ $4 = -4 + \beta 2^0$

Given that at $t = 0$ $y_t = 4$

or, $4 = -4 + \beta 2^0$

or $\boxed{8 = \beta}$

$\therefore$ Ans $y_t = -4 + 8(2)^t$ (ans)

③ $y_t = -7y_{t-1} + 16$

$y_t + 7y_{t-1} = 16$

PI : $\boxed{1\bar{y} + 7\bar{y}} = 16$ ✓

$8\bar{y} = 16$ ✓ $\therefore \boxed{\bar{y} = 2}$

$y_t = \beta$

CF:

$y_t + 7y_{t-1} = 0$

Let $y_t = \beta x^t$ trial solution.

$\therefore \beta x^t + 7\beta x^{t-1} = 0$

$\beta x^{t-1}[x+7] = 0$

$\boxed{x = -7}$

$\therefore \boxed{y_c = \beta(-7)^t}$

$\therefore y_t = PI + CF = \boxed{2 + \beta(-7)^t}$

$y_0 = 5$ ie at $t = 0$ $y_0 = 5$

or, $2 + \beta(-7)^0 = 5$

$\beta = 5 - 2 = 3$

$\therefore \boxed{y_t = 2 + 3(-7)^t}$ (ans)

$y_t - 4y_{t-1} = -6$

$\bar{y} - 4\bar{y} = -6$

$-3\bar{y} = -6$

$\boxed{\bar{y} = 2}$