

9862795123

Group & Ring Fields
Abelian Groups



field is most Structured F

$G \rightarrow$ most basic algebraic structure

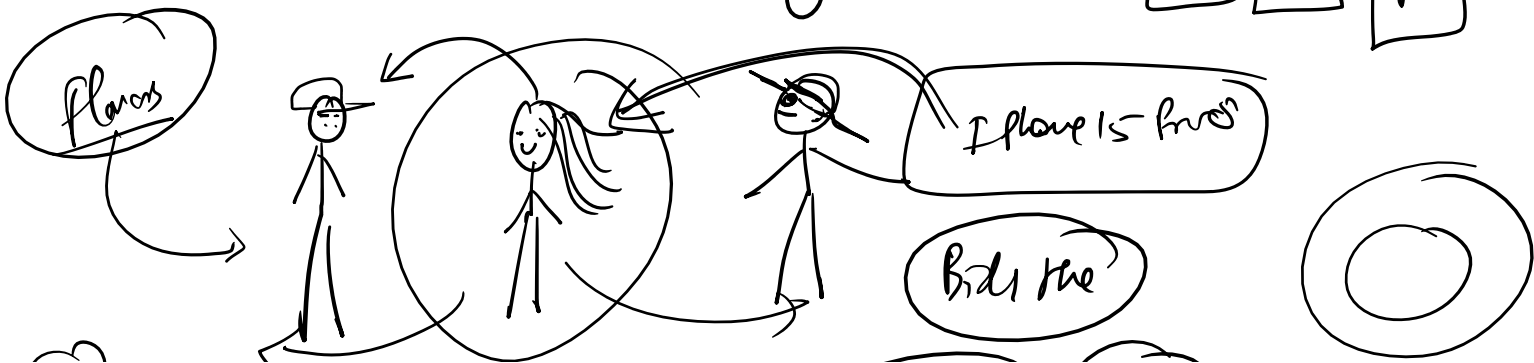
$R \rightarrow G + (X)$

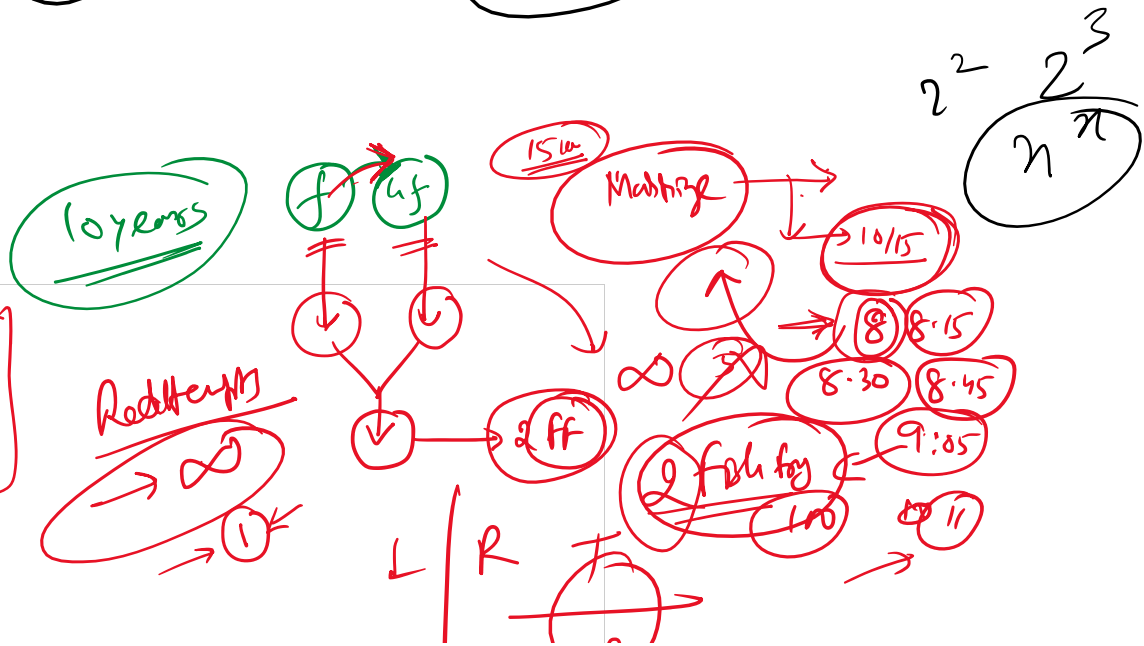
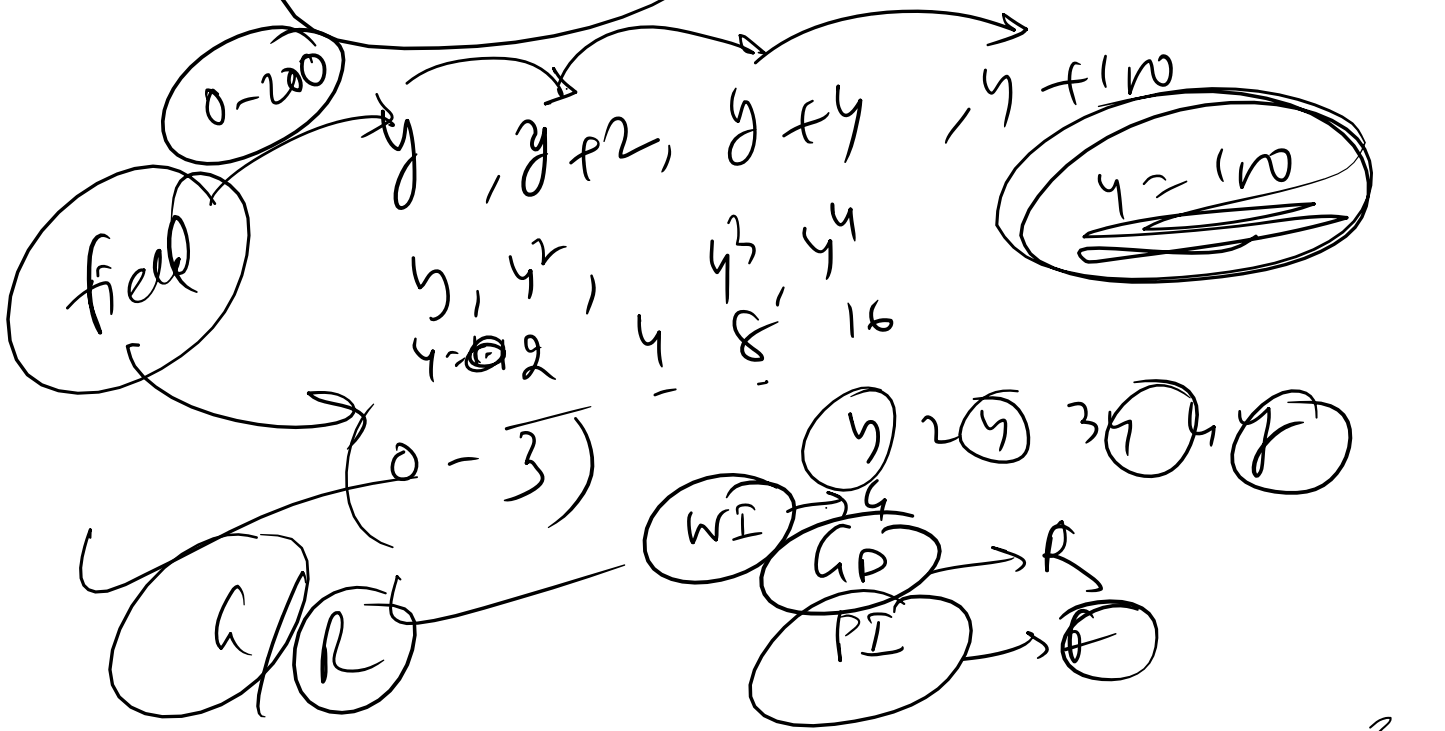
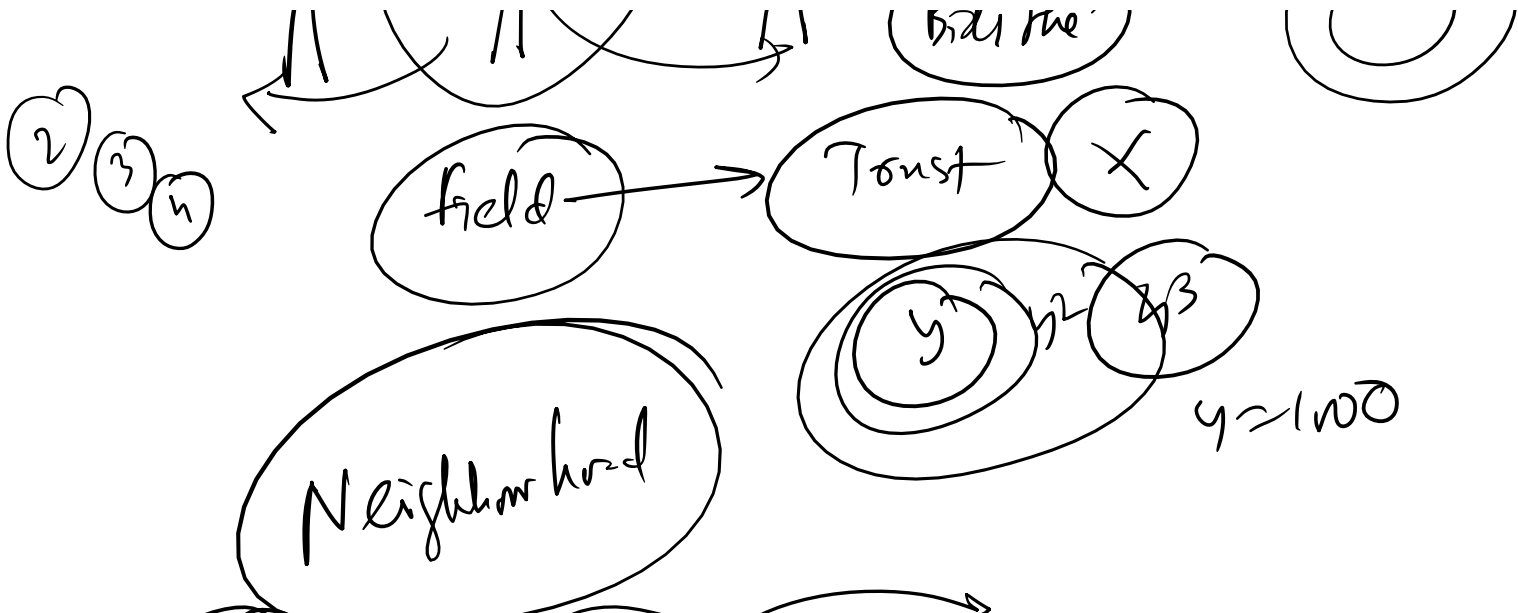
$F \rightarrow R + (\text{quotient of } X)$

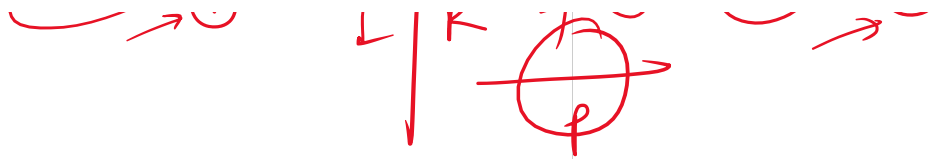
Every ring is a group under addition

Every field is a ring

Not all ring \rightarrow fields







Equivalence Relation

$4 \in \{0, \dots\}$
 $4 \in \{1, 2, 3, 4\}$
 $\mathcal{P}_{1,2} = \{ \emptyset, \{1\}, \{2\}, \{1,2\} \}$
 $\mathcal{P}_{1,2,3} = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$
 $\mathcal{P}_{1,2,3,4}$
 $\Rightarrow 2^4 - 1 = 15$

1, 2, 3, 4

Reflexive (a, a)
Symmetric $(a, b) \Rightarrow (b, a)$
Transitive $(a, b) \wedge (b, c) \Rightarrow (a, c)$

Antisymmetry $(a, b) \wedge (b, a) \Rightarrow a = b$
Asymmetry $(a, b) \Rightarrow \neg (b, a)$

2^{2n+3}

1. Let A be any set containing $2n+3$ elements, then the number of subsets having more than $n+1$ elements are
 (a) 2^{2n} (b) 2^{n-1}
 (c) $2^{2(n+1)}$ (d) 2^n

2. Let A and B be any two sets having 11 elements in common, then the number of elements common in $A \times B$ and $B \times A$ is

(a) 11 (b) $11^2 = 121$
 (c) 10 (d) 2^{11}

3. The Relation defined on the set $A = \{-3, -2, 1, 2\}$ by $R = \{(x, y) : |x-y| \leq 5\}$ is given by
 (a) $\{(-3, -2), (-3, 1), (-3, 2), (-2, 1)\}$
 (b) $\{(-2, 1), (-3, -2), (1, 2)\}$
 (c) $\{(2, 2), (1, 1), (1, -3), (-2, -3)\}$
 (d) None of these

4. Choose the correct statement:
 (a) Every reflexive relation on a set A is Symmetric.
 (b) Every reflexive relation on a set A is always anti symmetric.
 (c) Empty set on any set is always an equivalence relation.
 (d) Every Asymmetric relation is reflexive.

5. If A is any non-empty set with the cardinality "n" then the number of Asymmetric relation is
 (a) 2^n (b) 3^n
 (c) $3^{n(n-1)/2}$ (d) 3^{n-1}

6. Let $A = \{1, 2\}$, then Relation $S = \{(1, 1), (1, 2)\}$ is
 (a) Reflexive \checkmark
 (b) Irreflexive \checkmark
 (c) Symmetric \checkmark
 (d) Neither reflexive nor irreflexive \checkmark

$$\begin{aligned}
 2^{2n+3} &= C_0^{2n+3} + C_1^{2n+3} + \dots + C_n^{2n+3} + C_{n+1}^{2n+3} + \dots + C_{2n+3}^{2n+3} \\
 nCr &= nC(n-r) \\
 2^{2n+3} &= 2(C_{n+1}^{2n+3} + C_{n+2}^{2n+3} + \dots) \\
 2^{2n+3} &= 2 \cdot 2^{2n+2} \\
 2 &= 2^{2n+2}
 \end{aligned}$$

$$\begin{aligned}
 |A \cap B| &= 11 \\
 |(A \times B) \cap (B \times A)| &= |A \cap B| \times |B \cap A| \\
 &= 11 \times 11 = 121
 \end{aligned}$$

$$3^{n(n-1)/2}$$

- (b) Irreflexive ✓
- (c) Symmetric ✓
- (d) Neither reflexive nor irreflexive ✓

7. A nonempty relation which is reflexive relation cannot be

- (a) Symmetric
- (b) Asymmetric
- (c) Transitive
- (d) None of these

4

8. If $|A|=5$, then the number of equivalence relation is

- (a) 25
- (b) 52
- (c) 50

(b) 52 → definitely reflexive

9. The relation "greater than" on the set of natural number is

- (a) Only reflexive
- (b) Only Symmetric
- (c) Only transitive
- (d) Equivalence relation

10. For real number x and y , we define a relation $x R y \Leftrightarrow x-y + \sqrt{5}$ is an irrational number. Then the relation R is

- (a) Reflexive
- (b) Symmetric
- (c) Transitive
- (d) Equivalence relation

11. Quotient set of an uncountable set cannot be

- (a) Finite
- (b) Countable
- (c) Uncountable
- (d) None of these

12. Which of the following is an equivalence relation on the set of integers

- (a) $mRn \Leftrightarrow mn \geq 0$
- (b) $mRn \Leftrightarrow mn > 0$
- (c) $mRn \Leftrightarrow |m| = |n|$
- (d) $mRn \Leftrightarrow mn \leq 0$

13. Which of the following statement is true?

- (a) Intersection of two equivalence relation on a set is again an Equivalence relation.
- (b) Inverse of an equivalence relation is not an equivalence relation. $R \subset (A \times A)$, $R^{-1} = \{(y, x) : (x, y) \in R\}$
- (c) Union of two equivalence relation is again an equivalence relation.
- (d) None of these

14. Let Z , be the set of integers. Define a R b if and only if " $a + b$ " is a multiple of 5. Then relation is

- (a) Reflexive
- (b) Symmetric
- (c) Transitive
- (d) Not an equivalence relation

15. The number of not onto map from a set A of 5 elements to a set B of 3 elements

- (a) 120
- (b) 125
- (c) 100
- (d) 135

16. Let $A = \{2,3,4,5\}$ and Let $R = \{(2,2), (3,3), (4,4), (5,4)\}$ be a relation on A . Then R is

- (a) Reflexive
- (b) Symmetric
- (c) Transitive
- (d) None of these

$5 \times 0 = 0 + 5$
 $0 + 5 = 5 + 0$
 $1 + 5 = 5 + 1$
 $5 \times 0 \rightarrow 0$
 $a \times b \rightarrow a$

1 is not related to 6
 4 related to 4
 4 related to 6 but 1 is not related to 6

map of A to $B \rightarrow 3^5$

not onto
 $5C_1 (5-1)^3 + 5C_2 (5-2)^3 + 5C_3 (5-3)^3 + 5C_4 (5-4)^3$
 $= 5(64) + 10(27) + 10(8) + 5(1)$

Chapter - 4 Group theory

17. Let A be a set contains "15" elements, then the number of subsets of A having exactly "11" elements is
 (a) 1100 (b) 1165
 (c) 1253 (d) 1365
18. Let A be any set, then the power set of A cannot be
 (a) Finite (b) Countably infinite
 (c) Uncountable (d) Empty
19. Empty relation on an empty set is
 (a) Only Reflexive
 (b) Only Irreflexive
 (c) Neither reflexive nor irreflexive
 (d) Both reflexive and irreflexive ✓✓
20. If R is an equivalence relation on A and a, b ∈ A, then which of the following is possible
 (a) $Cl(a) \cap Cl(b) \neq \emptyset$ but $Cl(a) \neq Cl(b)$
 (b) $Cl(a) \cap Cl(b) = \emptyset$
 (c) $Cl(a) \cap Cl(b) = \{0\}$
 (d) None of these
21. If there are "m" number of equivalence relation on any set A, then the number of partition of set A is
 (a) m^2 (b) $2m$
 (c) m (d) $3m$
22. Let P be any set and $x \in P$, then $Cl(x)$ with any equivalence relation cannot be
 (a) Empty
 (b) Equal to the whole set P
 (c) Equal to x
 (d) None of these
23. Minimum number of transitive relation on a set whose cardinality is "n" is equal to
 (a) Number of Reflexive relation always
 (b) 5n
 (c) n^2
 (d) Number of partition of n
24. Define a relation ~ on N (Set of natural numbers) as $a \sim b \Leftrightarrow a/b \in Z$ (set of integers) $\forall a, b \in N$
 (a) Reflexive
 (b) Symmetric
 (c) Transitive
 (d) None of these

15C11

Power set ≠ Empty

1 17

→ 21 ←

element itself

X always takes $Cl(x)$

✓

Every equivalence relation → Transitive = Partitions