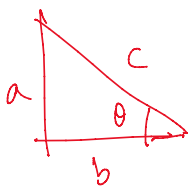
03 February 2024 17:02

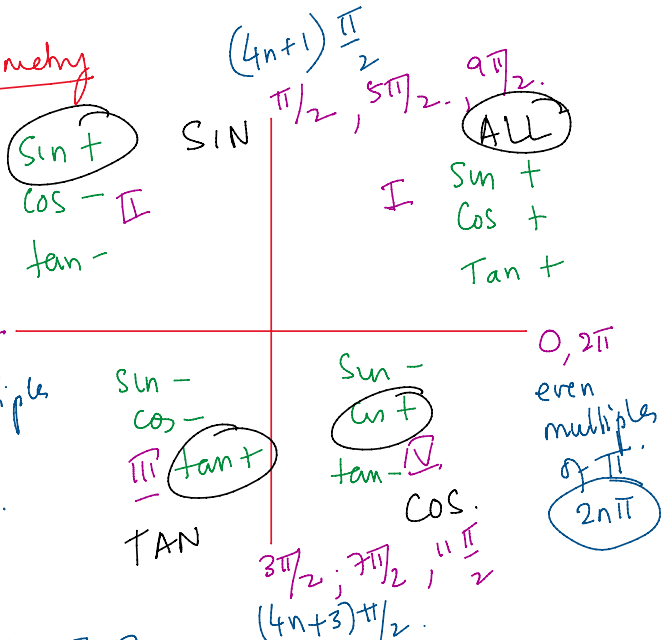
Trigono metry



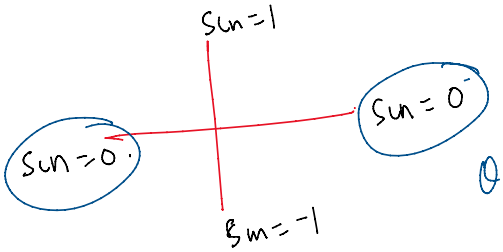
$\sin\theta = a/c$

$\cos\theta = b/c$

$\tan\theta = a/b$.



$\sin\theta = 0$

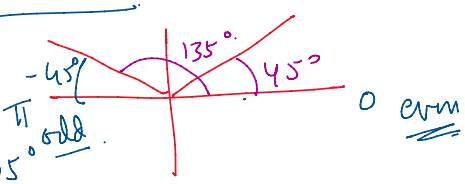


θ = even mult of π
or odd " " π } ⇒ for any multiple of π.

$\theta = n\pi$ → generic / general soln

$\theta = 0$ → particular soln

$\sin\theta = \frac{1}{\sqrt{2}}$. $\theta = ?$



$\theta = 0 + 45^\circ$ or $\pi - 45^\circ$

↓ $0 + 45^\circ + 2n\pi$ ↓ $\pi - 45^\circ + 2n\pi$

$\theta = n\pi + (-1)^n \left(\frac{\pi}{4}\right)$

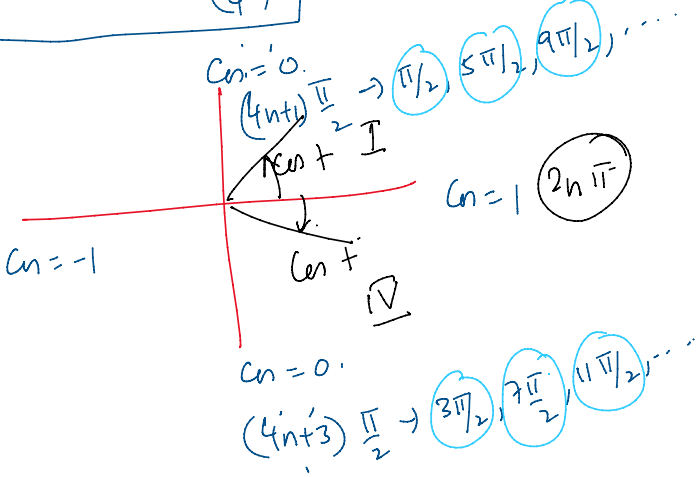
$\cos\theta = 0$.

θ = odd multiple of $\pi/2$

$= (2n+1)\pi/2$.

$\cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$.

$\theta = 2n\pi \pm \frac{\pi}{3}$.



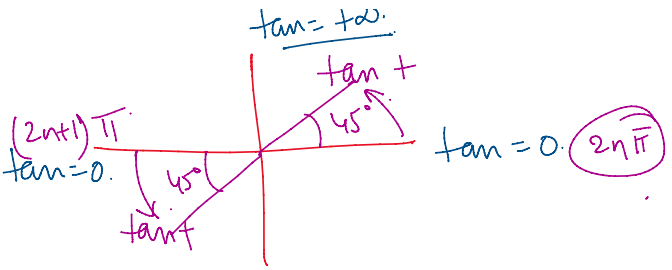
tan = ±∞.

tan +

$\tan\theta = 0$

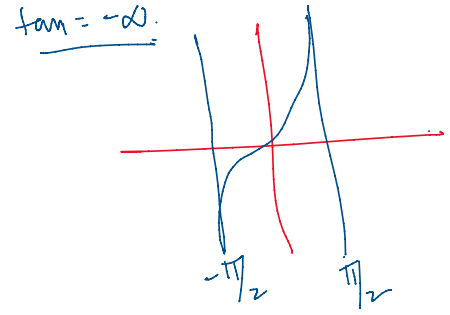
$\Rightarrow$ same as $\sin\theta = 0$.

$\theta = n\pi$



$\tan\theta = 1 = \tan\frac{\pi}{4}$

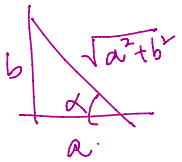
$\theta = n\pi + \frac{\pi}{4}$



$\sin\theta + \cos\theta = 1$

$a = 1 \quad b = 1$

$\sqrt{2}\sin(\alpha+\theta) = 1$



$\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$

$\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$

$\sin\alpha = \frac{1}{\sqrt{2}} = \cos\alpha,$

$\alpha = \frac{\pi}{4}$

$\cos(\theta + \pi/4) = \frac{1}{\sqrt{2}}.$

$\sin(\theta + \pi/4) = \sin\left(\frac{\pi}{4}\right)$

$\theta + \pi/4 = n\pi + (-1)^n \frac{\pi}{4}.$

$\theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$

$\theta = n\pi + \frac{\pi}{4}[(-1)^n - 1]$

To solve

$a\sin\theta + b\cos\theta$

Multiply and divide by $\sqrt{a^2+b^2}$

$= \sqrt{a^2+b^2}\left[\frac{a}{\sqrt{a^2+b^2}}\sin\theta + \frac{b}{\sqrt{a^2+b^2}}\cos\theta\right]$

$= \sqrt{a^2+b^2}\left[\cos\alpha\sin\theta + \sin\alpha\cos\theta\right]$

$= \sqrt{a^2+b^2}\sin(\alpha+\theta)$

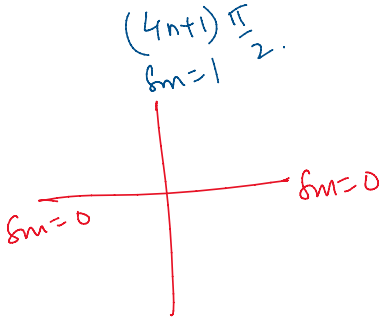
$2\sin^2\theta - 3\sin\theta + 1 = 0.$

$2\sin^2\theta - 2\sin\theta - \sin\theta + 1 = 0$

$2\sin\theta(\sin\theta - 1) - 1(\sin\theta - 1) = 0$

$(\sin\theta - 1)(2\sin\theta - 1) = 0$

$\sin\theta = 1, \frac{1}{2}$



$\sin\theta = 1$

$\sin\theta = \frac{1}{2}.$

$\sin\theta = -1$

$\sin\theta = 1$

$\theta = (4n+1)\frac{\pi}{2}$.

$\theta = 2n\pi + \frac{\pi}{2}$.

$\sin\theta = \frac{1}{2}$.

$\theta = n\pi + (-1)^n \frac{\pi}{6}$.

$\theta = n\pi + (-1)^n \frac{\pi}{6}$.

$2n\pi + \pi/2$, $2n\pi + \frac{\pi}{6}$, $(2n+1)\pi - \frac{\pi}{6}$.

$-1 \le \sin\theta \le 1$

$2\sin^2\theta + 2\sin\theta - 3 = 0$.

$D = 4 + 24 = 28$.

$\sin\theta = \frac{-2 \pm 2\sqrt{7}}{4} = \frac{-1 \pm \sqrt{7}}{2}$.

$\sqrt{7} = 2.6$

$\frac{-1+2.6}{2} = 0.8$ ✓

$\frac{-1-2.6}{2} = -1.8$

$\sin\theta = \frac{\sqrt{7}-1}{2}$.

$\theta = n\pi + (-1)^n \left[\sin^{-1}\left(\frac{\sqrt{7}-1}{2}\right)\right]$

$(-1)^2 = (1)^2$ ✓

$1 = 1$ ✓

$\sin\theta + \cos\theta = 1$.

$\sin\pi + \cos\pi = -1$

$\cos\theta = 1 - \sin\theta$

$\sqrt{1-\sin^2\theta} = 1 - \sin\theta$.

$1 - \sin^2\theta = 1 + \sin^2\theta - 2\sin\theta$.

$2\sin^2\theta - 2\sin\theta = 0$

$\sin\theta(\sin\theta - 1) = 0$.

$\sin\theta = 0, 1 \rightarrow \theta = n\pi, (4n+1)\frac{\pi}{2}$.

$\sqrt{2}\sin(\theta + \pi/4) = 1$.

$\sin(\theta + \pi/4) = \frac{1}{\sqrt{2}}$.

$\theta + \pi/4 = n\pi + (-1)^n \frac{\pi}{4}$.

for $n=0$ $\theta = 0$.

for $n=1$ $\frac{\pi}{2} = \theta$

$$\int \frac{dx}{\sin x + \cos x} = \int \frac{dx}{\sqrt{2}\sin(x+\pi/4)}$$

$$= \frac{1}{\sqrt{2}} \int \operatorname{cosec}(x+\pi/4)\, dx$$

1. $\sin\theta = 0 \Leftrightarrow \theta = n\pi$
2. $\cos\theta = 0 \Leftrightarrow \theta = (2n+1)\dfrac{\pi}{2}$
3. $\tan\theta = 0 \Leftrightarrow \theta = n\pi$
4. $\sin\theta = \sin\alpha \Leftrightarrow \theta = n\pi + (-1)^n\,\alpha$, where $\alpha \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$
5. $\cos\theta = \cos\alpha \Leftrightarrow \theta = 2n\pi \pm \alpha$, where $\alpha \in [0, \pi]$
6. $\tan\theta = \tan\alpha \Leftrightarrow \theta = n\pi + \alpha$, where $\alpha \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$\sin^2\theta = \sin^2\alpha$.

Whenever you have $\sin^2\theta$ or $\cos^2\theta$ use $\cos 2\theta$ →

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 2\sin^2\alpha$.

$1 - \cos 2\theta = 1 - \cos 2\alpha$.

$\cos 2\theta = \cos 2\alpha$

$2\theta = 2n\pi \pm 2\alpha$

$(\sin\theta - \sin\alpha)(\sin\theta + \sin\alpha) = 0$

$2\sin\left(\frac{\theta-\alpha}{2}\right)\cos\left(\frac{\theta+\alpha}{2}\right)$

$\cdot 2\sin\left(\frac{\theta+\alpha}{2}\right)\cos\left(\frac{\theta-\alpha}{2}\right)$

$= 0.$