18 November 2023 07:32 PM

Q1 Evaluate the probability function from the following distribution function of a discrete variable x

$F(x) = 0$	$x < 0$
$= 1/8$	$0 \le x < 2$
$= 2/8$	$2 \le x < 3$
$= 5/8$	$3 \le x < 4$
$= 1$	$x \geq 4$

$$F(x) = P(X \le x) = \sum_{y \le x} f(y)$$

For the given distribution function it is clear that $f(x) > 0$ for $x = 0, 2, 3, 4$

$$f(0) = \frac{1}{8} - 0 = \frac{1}{8}$$

$$f(2) = 2/8 - \frac{1}{8} = \frac{1}{8}$$

$$f(3) = \frac{5}{8} - \frac{2}{8} = \frac{3}{8}$$

$$f(4) = 1 - 5/8 = \frac{3}{8}$$

18   8

Thus the probability function is

$$f(x) = \frac{1}{8} \quad \text{for } x = 0$$
$$= \frac{1}{8} \quad \text{for } x = 2$$
$$= 3/8 \quad \text{for } x = 3$$
$$= 3/8 \quad \text{for } x = 4$$
$$= 0 \quad \text{elsewhere.}$$

Q2 The probability mass function $f(x)$ of a random variable X is 0 except at the points

$x = 0, 1, 2$

and $f(0) = c$, $f(1) = 2c - 3c^2$

$f(2) = 4c - 1$

(i) Determine the value of c.

(ii) find $P(X > 0 \mid X < 2)$.

(iii) Find expectation and variance of X.

(i) Since $f(x)$ is a pmf then

and $\sum_x f(x) = 1$ $f(x) \geq 0$ for all x

or, $f(0) + f(1) + f(2) = 1$

or, $c + 2c - 3c^2 + 4c - 1 = 1$

or, $3c^2 - 7c + 2 = 0$

or, $3c^2 - 6c - c + 2 = 0$

or, $3c(c-2) - 1(c-2) = 0$

or, $(3c-1)(c-2) = 0$

$\therefore c = \frac{1}{3}$ or 2

But $c \neq 2$, because $f(0) = c = 2 > 1$ (Not possible)

So $c = \frac{1}{3}$

(ii) $f(0) = \frac{1}{3}$ at $x = 0$

$f(1) = 2c - 3c^2 = 2\times\frac{1}{3} - \cancel{3}\times\frac{1}{\cancel{9}3} = \frac{1}{3}$ at $x = 1$

$f(2) = 4c - 1 = 4\times\frac{1}{3} - 1 = 1/3$ at $x = 2$

We have to find $P(x > 0 \mid x < 2)$

$$= \frac{P[(x>0) \cap (x<2)]}{P(x<2)}$$

$$= \frac{P(x=1)}{P(x=0) + P(x=1)} = \frac{1/3}{1/3 + 1/3}$$

$= 1/2$ (ans)

(iii) E(x) and V(x)

$$E(x) = \sum_x x \cdot f(x)$$
$$= \sum_{x=0}^{2} x \cdot f(x)$$
$$= 0 \times f(0) + 1 \times f(1) + 2 \times f(2)$$
$$= 0 + 1 \times 1/3 + 2 \times 1/3$$
$$= \frac{1}{3} + \frac{2}{3}$$
$$= 1 \text{ (ans)}$$

$$V(x) = E(x^2) - [E(x)]^2$$
$$E(x^2) = 0^2 \times f(0) + 1^2 \times f(1) + 2^2 \times f(2)$$
$$= 0 + \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$$
$$V(x) = \frac{5}{3} - 1 = 2/3 \text{ (ans)}$$

3: Is the following a probability density function? pdf

check whether f(x) is continuous or not pdf ? X probability

$$f(x) = 2x, \quad 0 < x \le 1$$
$$= 4 - 2x, \quad 1 < x \le 2$$
$$= 0, \quad \text{elsewhere}$$

Ans For f(x) to be a pdf, it has to satisfy

(i) $f(x) \geq 0$ for all x

✓ (i) $f(x) \geq 0$ for all x

✓ (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Here $f(x) \geq 0$ for all x

ie for $0 < x \leq 1$; $f(x) = 2x > 0$

for $1 < x \leq 2$: $f(x) = 4 - 2x > 0$

Again $\int_{-\infty}^{\infty} f(x)\,dx = \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx$

$$= \int_{0}^{1} 2x\,dx + \int_{1}^{2} (4-2x)\,dx$$

$$= [x^2]_0^1 + 4[x]_1^2 - [x^2]_1^2$$

$$= 1 + 4(1) - 3$$

$$= 1 + 1$$

$$= 2$$

$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 2 \neq 1$

$\therefore$ $f(x)$ is not a pdf.

4. A continuous r.v. X has a density function given by

$$f(x) = \frac{1}{2} - ax, \quad 0 \le x \le 4$$
$$= 0 \quad \text{elsewhere}$$

where 'a' is constant.

Find (i) the value of 'a'.
(ii) $P(1 < x < 2)$ (iii) $P(2x + 3 > 5)$
and (iv) $E(x)$

(i) Since $f(x)$ is a p.d.f.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

or, $\displaystyle\int_0^4 \left(\frac{1}{2} - ax\right) dx = 1$

or, $\frac{1}{2}[x]_0^4 - \frac{a}{2}[x^2]_0^4 = 1$

or, $\frac{1}{\cancel{2}} \times \cancel{4}^{2} - \frac{a}{\cancel{2}} \times \cancel{16}^{8} = 1$

$\Rightarrow 2 - 8a = 1$

or, $8a = 1$

or, $\boxed{a = 1/8}$ ans.

or, $\boxed{a = 1/8}$ ans.

i) pdf is $f(x) = \frac{1}{2} - \frac{x}{8}$; $0 \leq x \leq 4$
$= 0$ elsewhere

(ii) $P(1 < x < 2) = \int_1^2 f(x)\,dx$

$= \left[\frac{x}{2} - \frac{x^2}{16} \right]_1^2 = 5/16$ (ans)

(iii) $P(2x+3 > 5)$
$= P(x > 1) = \int_1^4 (f(x))\,dx$

$= \int_1^4 \left(\frac{1}{2} - \frac{x}{8} \right) dx$

$= \frac{1}{2}[x]_1^4 - \frac{1}{4}[x^2]_1^4$

$= 9/16.$

(iv) $E(x) = \int^{\infty} x\, f(x)\,dx$

$$= \int_{0}^{4} x\left(\frac{1}{2}-\frac{x}{8}\right)dx$$

$$= \int_{0}^{4} \frac{x}{2}\,dx - \frac{1}{8}\int_{0}^{4} x^2\,dx$$

$$= \left[\frac{x^2}{4}\right]_0^4 - \frac{1}{8\times 3}\left[x^3\right]_0^4$$

$$= \frac{4^2}{4} - \frac{1}{24}\left(4^3\right)$$

$$= 4 - \frac{1}{6}\times 4^2$$

$$= 4\left[1-\frac{4}{6}\right]$$

$$= 4\times\frac{1}{3} = 4/3 \text{ (ans)}.$$

(5) For the following probability distribution check whether x is a continuous r.v. and find mean and s.d of x.

Check wa...

Also find mean and s.d of x.

pdf? $f(x) = \frac{4x}{5}$, $0 < x \le 1$

$= \frac{2}{5}(3-x)$, $1 < x \le 2$

$= 0$, elsewhere.

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 \frac{4x}{5}dx + \int_1^2 \frac{2}{5}(3-x)dx$

$= \frac{4}{5}\left[\frac{x^2}{2}\right]_0^1 + \frac{2}{5}\left[3x - \frac{x^2}{2}\right]_1^2$

$= \frac{4}{5}\left(\frac{1}{2}\right) + \frac{2}{5}\left[6 - 3 - \frac{4}{2} + \frac{1}{2}\right]$

$= \frac{4}{5} \times \frac{1}{2} + \frac{2}{5}\left[3 - \frac{3}{2}\right]$

$= \frac{4}{10} + \frac{2}{10}[3]$

$= \frac{4}{10} + \frac{6}{10} = \frac{10}{10} = 1$

$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$

$\therefore f(x)$ is a pdf

ie x is a cont. r.v.

mean $E(x) = 17/15$ and

s.d $= \sqrt{V(x)} = \frac{\sqrt{194}}{30}$

(mean … /15

$s.d = \sqrt{V(x)} = \frac{\sqrt{194}}{30}$)