Thursday, November 2, 2023 6:38 PM

STATISTICS
Wirelland Onhillon d CL7







Statistics Questions

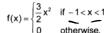
# SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

- An unknown distribution has a mean of 90 and a standard deviation of 15. Samples of size n = 25 are drawn randomly from the population. Find the probability that the sample mean is between 85 and
  - 0.3546 (A)
  - 0.1254 (C)
- 0.6997 0.4521
- A produce company claims that the mean weight of peaches in a large shipment is 6.0 oz with a standard deviation of 1.0 oz. Assuming this claim is true, what is the probability that a random sample of 1900 of these peaches would have a mean weight of 5.9 oz or less?
  - 0.0008

0.654

6.0 (D)

Let Y = X<sub>1</sub> + X<sub>2</sub> +... + X<sub>15</sub> be the sum of a random sample of size 15 from the distribution whose density function is



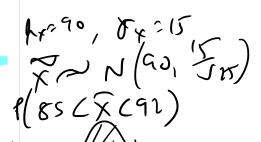
 $\label{eq:local_problem} \begin{picture}(100,0) \put(0,0){\line(1,0){100}} \put(0,0){\line(1,0){10$ 

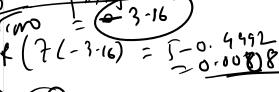
(A) 0.2131

(B) 0.5214

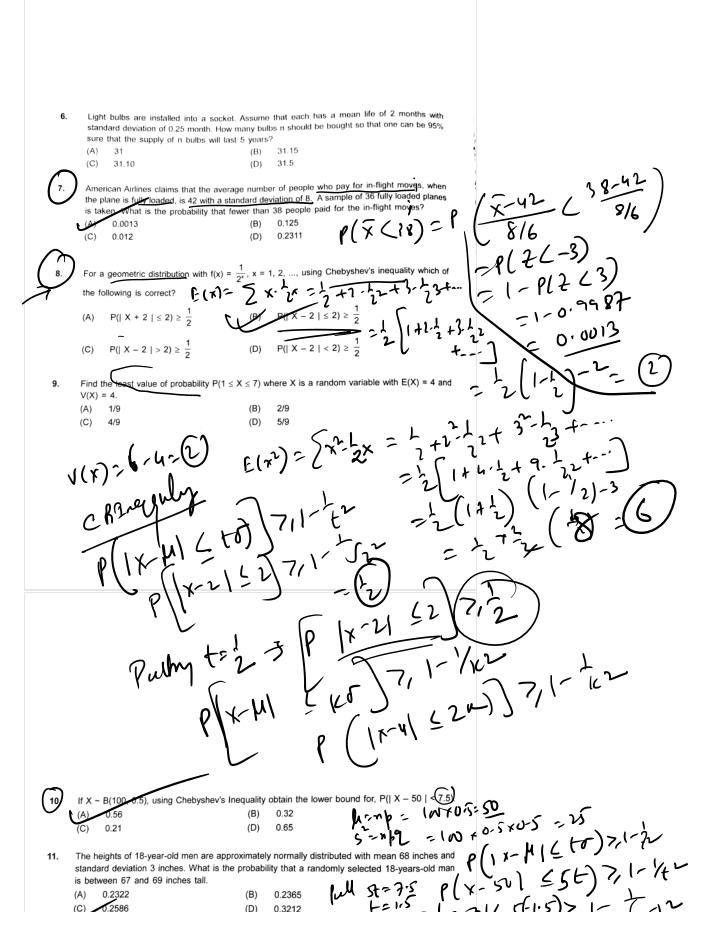
(C) 0.2313

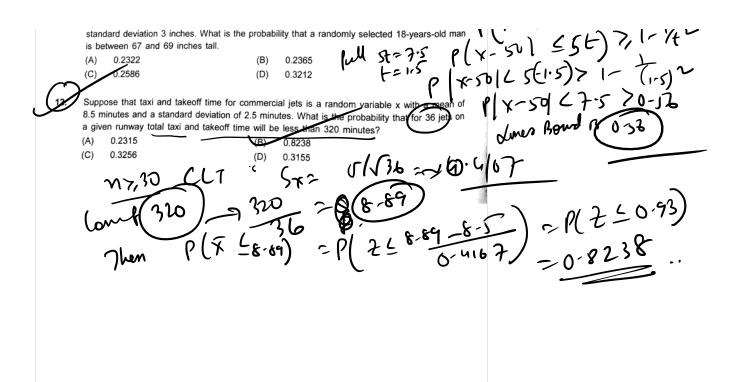
(D) None of the above

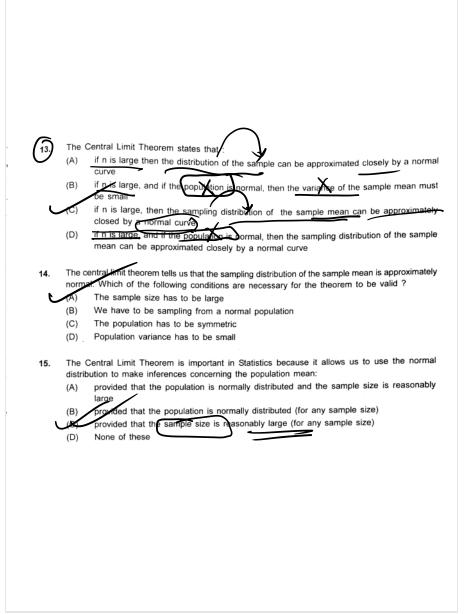




5.	μ = 71. sample (A) (C) Light b	43 and variance mean is betwe 0.3654 0.3654	$e^{-\sigma^2} = 56.25$ . Let seen 68.91 and 71	X be the .97? (B) (D)	n = 25 from a popul sample mean. What is 0.21465 0.5941 If we assume that ear months, what is the 0.57 None of the above	is the probability to	that the







9062395723

- The Central Limit Theorem is important in Statistics because :
  - it tells us that large samples do not need to be selected
  - it guarantees that, when it applies, the sample that are drawn are always randomly selected

it enables reasonably accurate probabilities to be determined for events involving the sample average when the sample size is large regardless of the distribution of the

- (D) it tells us that if several samples have product sample averages which seem to be different than expected, the next sample average will likely be close to its expected value
- A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes

19/20 19/24

24/19 (D)

E(5)= 1 b = 100

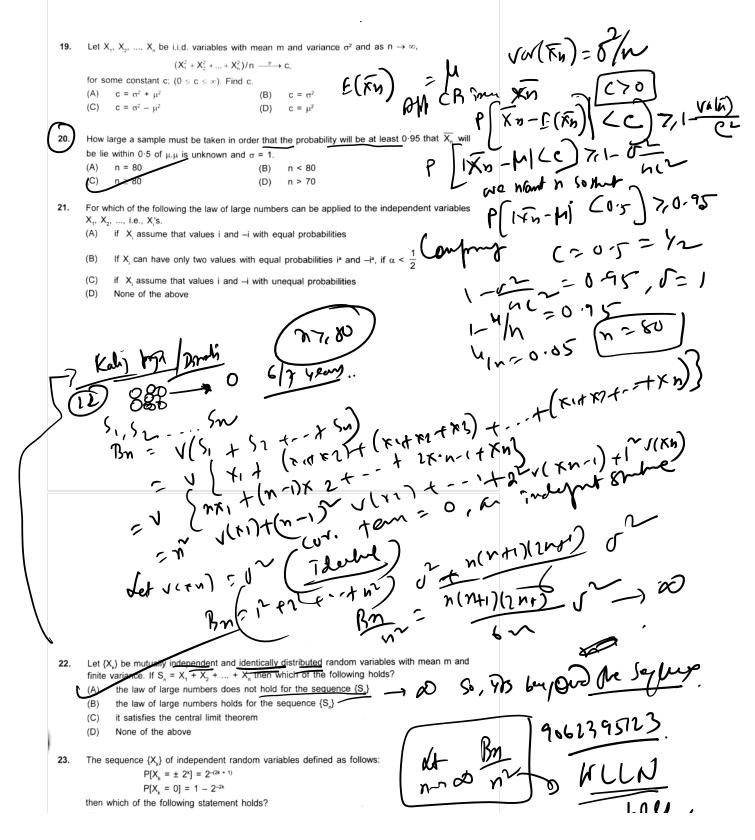
Use Chebychev's inequality to determine how many times a fair coin must be tossed in order that the probability will be at least 0.90 that the ratio of the observed number of heads to the number of tosses will lie between 0.4 and 0.6.

300

250

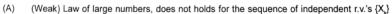
(D) 255

New Section 1 Page 6



$$P[X_k = \pm 2^k] = 2^{-(2k+1)}$$
  
 $P[X_k = 0] = 1 - 2^{-2k}$ 

then which of the following statement holds?



- (B) (Weak) Law of large numbers, holds for the sequence of independent r.v.'s  $\{X_k\}$
- (C) (Strong) of large numbers, holds for the sequence of independent r.v.'s  $\{X_k\}$
- (D) None of the above
- 24. Let  $X_1, X_2, ..., X_n$  be jointly normal with  $E(X_i) = 0$  and  $E(X_i^2) = 1$  for all i and  $Cov(X_i, X_j) = \rho$  if |j-i| = 1 and 0 otherwise, then which of the following statement holds ?
  - (A) WLLN does not hold for the sequence  $|X_n|$
  - (B) SLLN holds for the sequence  $|X_n|$
  - (C) WLLN holds for the sequence |X<sub>n</sub>|
  - (D) None of the above

indent r.v.'s {X<sub>s</sub>}

r.v.'s {X<sub>s</sub>}

{X<sub>s</sub>}

- 25. Suppose that some distribution with an unknown mean has variance equal to 1. How large a sample must be taken in order that the probability will be at least .95 that the sample mean  $\overline{X}_{\rm n}$ will lie within .5 of the population mean?
  - (A) 20

(B) 63

(C) 51

- (D) 80
- **26.** Let  $\{X_n, n \ge 1\}$  be i.i.d. N(0, 1) then

$$cos \Biggl( \frac{X_1^4 + X_2^4 + \ldots + X_n^4}{n} \Biggr)$$

as  $n \to \infty$ ?

(A) cos(2) (C) cos(3)

- (B) cos(5) (D) None of the above
- 27. Let  $\{X_n, \ n \geq 1\}$  be i.i.d. with mean  $\mu$  and finite variance  $\sigma^2$ . Let

$$\mu_n := \frac{X_1 + ... + X_n}{n} \text{ and } \sigma_n^2 = \frac{(X_1 - \mu_n)^2 + ... + (X_n - \mu_n)^2}{n} \,.$$

then

28. Let  $\{X_n, n \ge 1\}$  be a sequence of independent Bernoulli random variables with mean  $p \in (0, 1)$ . We construct an estimate  $\hat{p}_n$  of p from  $\{X_1,\,...,\,X_n\}.$  We know that  $p\in(0.4,\,0.6).$  Find the smallest value of n so that

$$P\!\left(\frac{\left|\hat{p}_n-p\right|}{p}\leq 5\%\right)\geq 95\%.$$

2000 (A)

(B) 2400

(C) 2600

- 2500 (D)
- Let  $\{\epsilon_{\!_R}\!\}$  be an iid sequence. A generic term of the sequence has mean  $\mu$  and variance  $\sigma^2.$  Let  $\{X_n\}$  be a covariance stationary sequence such that a generic term of the sequence satisfies

$$X_n = \rho X_{n-1} + \varepsilon_n$$

where  $-1 < \rho < 1$ . Denote by

$$\overline{X}_n = \frac{1}{n} \sum_{i=1}^n X_n$$

the sample mean of the sequence. Verify whether the sequence  $\{X_n\}$  satisfies the conditions that are required by Chebyshev's Weak Law of Large Numbers. In affirmative case, find its probability limit.

 $\begin{array}{ll} \text{(A)} & \frac{\mu}{1-\rho} \\ \text{(C)} & \frac{2\mu}{1-\rho} \end{array}$ 

(D) None of the above

- 30. Suppose X is B(100; 0.3). Compute P(X > 35).

(B) 0.1379

(C)

(D) 0.1236

# SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

Suppose the number X of items produced by a factory in a week is a random variable with mean 1.

Then which of the following are correct?

- (A) an upper bound for  $P(X \ge 25) = 0.75$
- an upper bound for P(X  $\geq$  75) =  $\frac{2}{3}$ a lower bound for P(40 < X < 60) = 0.75 (B)
- a lower bound for P(40 < X < 60) =  $\frac{2}{3}$ (D)
- Let X be a r.v. with cdf  $F_X(x)$  and pdf  $f_X(x)$ . Let Y = aX + b, where a and b are real contents and a ≠ 0.

Then which of the following holds?

- the cdf of Y in terms of  $F_\chi(x),$  is  $F_\gamma(y)$  = 1  $F_\chi\!\left(\frac{y-b}{a}\right)$  a < 0 (A)
- the pdf of Y in terms of  $f_\chi(x),$  is  $F_{_Y}(y)$  = 1  $F_\chi\!\left(\frac{y-b}{a}\right)$  a < 0 (B)
- the pdf of Y in terms of  $f_X(x)$ , is  $f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$ (C)
- the cdf of Y in terms of  $F_X(x)$ , is  $f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$

- Let  $Y = X^2$  then the pdf of Y if X = N(0; 1), will be
- (A)  $\frac{1}{\sqrt{y}} f_X(\sqrt{y}) = \frac{1}{\sqrt{2\pi y}} e^{-y/2} \quad y > 0$ (B)  $0 \quad y < 0$ (C)  $\frac{1}{\sqrt{y}} f_X(\sqrt{y}) = \frac{1}{\sqrt{2\pi y}} e^{-y/2} \quad y < 0$ (D)  $\frac{1}{\sqrt{y}} f_X(\sqrt{y}) = \frac{1}{\sqrt{2\pi y}} e^{-y/2} \quad y = 0$
- Let the moment of a discrete r.v. X be given by

$$E(X^k) = 0.8$$

- $E(X^k) = 0.8$  k = 1, 2, ... P(X = 1) = 0.2 P(X = 1) = 0.8(A) P(X = 1) = 0.2
  - (B) P(X = 0) = 0.2
- (C) P(X = 1) = 0.8
- (D) P(X = 0) = 0.8
- Light bulbs are installed into a socket. Assume that each has a mean life of 2 months with standard deviation of 0.25 month. How many bulbs n should be bought so that one can be 95% sure that the supply of n bulbs will last 5 years ?
  - (A) 36 (C) 30

(D) 31

- The ounce of fill from the bottling machine are assumed to have a normal distribution with  $\sigma^2$  = 1. Suppose that we plan to select a random sample of ten bottles and measure the amount of fill in each bottle. If these ten observations are used to calculate S2, it might be useful to specify an interval of values that will include S2 with a high probability. Then which of the following are correct?
  - (A) b<sub>1</sub> = 0.369
- (C) b<sub>1</sub> = 1.369
- (B)  $b_2 = 1.880$ (D)  $b_2 = 1.008$
- Let  $\mathbf{X_1}$  and  $\mathbf{X_2}$  be a random sample of size 2 from a distribution with probability density function

$$f(x) = \begin{cases} 6x(1-x) & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

for the sample sum  $Y = X_1 + X_2$ ? which of the following are correct?

- (A) mean (Y) = 0.5
- (B) Var (Y) = 0.05
- (C) Var (Y) = 0.1
- (D) mean (Y) = 1
- If  $X_1, X_2, ..., X_n$  are mutually independent random variables with respective means  $\mu_1, \ \mu_2, \ ..., \ \mu_n$  and variances  $\sigma_1^2, \sigma_2^2, ..., \sigma_n^2$ , then the mean and variance of Y =  $\sum_{i=1}^n a_i X_i$ , where  $a_i, a_2, ..., a_n$  are real constants, are given by
  - $(A) \qquad \sigma_Y^2 = \sum_{i=1}^n a_i \ \sigma_i^2$  $\begin{array}{lll} \text{(A)} & & \sigma_{Y}^{2} = \sum_{i=1}^{n} a_{i} \; \sigma_{i}^{2} & & \\ \text{(B)} & & \mu_{Y} = \sum_{i=1}^{n} a_{i} \, \mu_{i} \\ \\ \text{(C)} & & \sigma_{Y}^{2} = \sum_{i=1}^{n} a_{i}^{2} \, \sigma_{i}^{2} & & \\ \text{(D)} & & \mu_{Y} = \sum_{i=1}^{n} a_{i}^{2} \, \mu_{i} \end{array}$

- 9. Let the independent random variables  $X_1$  and  $X_2$  have means  $\mu_1$  = -4 and  $\mu_2$  = 3, respectively and variances  $\sigma_1^2$  = 4 and  $\sigma_2^2$  = 9. Then which of the following are correct?
  - (A) mean = -18
- (B) var = 27
- (C) mean = 72
- (D) var = 72
- 10. Let  $X_1$ ,  $X_2$ , ...,  $X_{50}$  be a random sample of size 50 from a distribution with density

$$f(x) = \begin{cases} \frac{1}{\theta} \, e^{-\frac{x}{\theta}} & \text{for } 0 \leq x < \infty \\ 0 & \text{otherwise}. \end{cases}$$

which of the following are correct?

- (A) mean =  $\theta$
- (B)  $\operatorname{var} = \frac{\theta^2}{50}$
- (C) mean =  $\frac{\theta^2}{50}$
- (D)  $var = \theta$

#### SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

- 1. There are 100 men on a plane. Let  $X_i$  be the weight (in pounds) of the ith man on the plane. Suppose that the  $X_i$ 's are i.i.d., and  $EX_i = \mu = 170$  and  $\sigma_X = \sigma = 30$ . The probability that the total weight of the men on the plane exceeds 18,000 pounds equals to \_\_\_\_\_\_.
- 2. Let  $X_1, X_2, ..., X_{25}$  be i.i.d. with the following PMF

$$P_{x}(k) = \begin{cases} 0.6 & k=1\\ 0.4 & k=-1\\ 0 & \text{otherwise} \end{cases}$$

And let

$$Y = X_1 + X_2 + ... + X_n$$

Using the CLT and continuity correction,P(4 ≤ Y ≤ 6) = \_\_\_\_\_

- 3. You have invited 64 guests to a party. You need to make sandwiches for the guests. You believe that a guest might need 0, 1 or 2 sandwiches with probabilities  $\frac{1}{4}, \frac{1}{2}$ , and  $\frac{1}{4}$  respectively. You assume that the number of sandwiches each guest needs is independent from other guests. Number of sandwiches you should make so that you are 95% sure that there is no shortage equals to
- 4. Let  $X_1$ ,  $X_2$ , ...,  $X_n$  be i.i.d. **Exponential(\lambda)** random variables with  $\lambda$  = 1. Let  $\overline{X} = \frac{X_1 + X_2 + ... + X_n}{n}$ .

If  $P(0.9 \le \overline{X} \le 1.1) \ge 0.95$  then n should be \_\_\_\_\_.

5.	A bank teller serves customers standing in the queue one by one. Suppose that the service time $X_i$ for customer i has mean $EX_i = 2$ (minutes) and $Var(X_i) = 1$ . We assume that service times for different bank customers are independent. Let $Y$ be the total time the bank teller spends serving 50 customers then $P(90 < Y < 110) = \underline{\hspace{1cm}}$ .
6.	In a communication system each data packet consists of 1000 bits. Due to the noise, each bit may be received in error with probability 0.1. It is assumed bit errors occur independently. The probability that there are more than 120 errors in a certain data packet equals to
7.	The probability of exactly 55 heads in 100 tosses of a coin equals to
8.	A coin is tossed 100 times. The probability that the number of heads lies between 40 and 6 (the word \between" in mathematics means inclusive of the endpoints) equals to
9.	A die is rolled 420 times. The probability that the sum of the rolls lies between 1400 and 155 equals to

10.	The value that is 2 standard deviations above the expected value (it is 90) of the sample mean equals to
11.	Customers arrive at a mall in accordance with a Poisson process with rate 4,000 persons per day. The approximate value for the probability that tomorrow at least 3850 customers will enter the mall equals to
12.	If a can of paint covers on the average 5/3.3 square feet with a standard deviation of 31.5 square feet, then the probability that the mean area covered by a sample of 40 of these cans will be anywhere from 510.0 to 520.0 square feet equals to
13.	A distribution with unknown mean $\mu$ has variance equal to 1.5. If we use central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be at least 0.95 that the sample mean will be within 0.5 of the population mean.
14.	If $X_1$ , $X_2$ ,, $X_n$ are poisson variates with parameter $\lambda$ = 2, use the central limit theorem to estimate P(120 $\leq$ S <sub>n</sub> $\leq$ 160), where S <sub>n</sub> = $X_1$ + $X_2$ ++ $X_n$ and n = 75.
15.	The new Endeavor SUV has been recalled because 5% of the cars experience brake failure. The Tahoe dealership has sold 200 of these cars. The probability that fewer than 4% of the cars from Tahoe experience brake failure equals to

16.	Take a random sample of size n = 15 from a distribution whose probability density function is:
	$f(x) = \frac{3}{2}x^2$
	for $-1 < x < 1$ . The probability that the sample mean falls between $-2/5$ and $1/5$ is
17.	The average time between Infection with the AIDS virus and developing AIDS has been estimated to be 8yr with a standard deviation of about 2yr. Approximately, what fraction of people develop ADS with in 4yr of infection ?

18. If  $\boldsymbol{X}_{1},\ \boldsymbol{X}_{2},\ ...$  are iid RVs with common mean  $\boldsymbol{\mu}$  and finite fourth moment, then

$$P\left\{\lim_{n\to\infty}\frac{S_n}{n}=\mu\right\}=\underline{\hspace{1cm}}$$

- 19. Let  $X_1, X_2, ...$ , be iid  $P(\lambda)$  RVs. and  $S_n$  has approximately in  $N(n\lambda, n\lambda)$  distribution for large n then  $P(S_n = 10) = \underline{\hspace{1cm}}$
- 20. Let X be a r.v. with pdf  $f_X(x)$ . Let Y =  $X^2$ . the pdf of Y for  $y \le 0$  equals to \_\_\_\_\_\_.

#### ANSWER KEY

#### SECTION-(A) MULTIPLE CHOICE QUESTIONS (MCQ)

1	2	3	4	5	6	7	8	9	10
В	Α	С	D	Α	В	Α	В	D	Α
11	12	13	14	15	16	17	18	19	20
С	В	С	Α	С	С	С	В	Α	С
21	22	23	24	25	26	27	28	29	30
В	Α	В	С	D	С	В	В	Α	В

### SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

1	2	3	4	5	6	7	8	9	10
B,C	A,C	A,B	B,C	В	A,B	C,D	B,C	A,D	A,B

### SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

1	2	3	4	5	6	7	8	9	10
0.0004	0.2405	74	385	0.8427	0.0175	0.484	0.9642	0.9670	96
11	12	13	14	15	16	17	18	19	20
0.9911	0.6553	16	0.7952	0.2090	0.8185	12	1	0.1009	0