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$y_t - 4y_{t-1} = -6$

$\bar{y} = 2$

CF: $y_t - 4y_{t-1} = 0$

Let the trial solution be $y_t = \beta x^t$

$y_{t-1} = \beta x^{t-1}$

$\therefore \beta x^t - 4\beta x^{t-1} = 0$

$\beta x^{t-1}(x-4) = 0$

$x - 4 = 0$

$\boxed{x = 4}$

$\therefore y_t = \beta 4^t$

$\therefore$ The required solution is

$y_t = PI + CF$

$$\boxed{y_t = 2 + \beta 4^t}$$

$\hat{C}, \hat{I}, \hat{Y}$

Income changes at a rate proportional to excess demand

$C(t) = a\, Y(t)$

$I(t) = b\, Y(t)$

$C + I - Y$

$$\frac{dY}{dt} \propto C+I-Y$$

$$\frac{dY}{dt} = a(C+I-Y)$$

$$\frac{dY}{dt} = a\left(gY(t) + bY(t) - Y(t)\right)$$

$$\frac{dY}{dt} = a(g+b-1)Y(t)$$

$$\frac{dY}{dt} - a(g+b-1)Y(t) = 0$$

↳ 1st order, homogenous differential equation with constant coefficient.

$$Y(t) = Ae^{-\int a(g+b-1)dt}$$

$$= Ae^{-a(g+b-1)t+c}$$

$$= Ae^{c}\, e^{-a(g+b-1)t}$$

$$Y(t) = \lambda e^{-a(g+b-1)t}$$

at $t=0$ $\boxed{Y(t) = Y(0) - Y_e}$

$$Y(0) - Y_e = \lambda e^{-a(g+b-1)\times 0}$$

$$\lambda = Y(0) - Y_e$$

$$\therefore \; Y(t) = [Y(0) - Y_e]\, e^{-a(g+b-1)t}$$

$t \to \infty$

$Y(t) = Y(0) - Y_e \; e^{a(g+b-1)t}$

$Y(t) \to Y_e$ as $g+b<1$

Domar's Model:

$P = Y$

productive capacity; agg demand

(1) initial supply = demand

(2) $$\frac{dP}{dt} = \frac{dY}{dt}$$

Rate of change in supply = Rate of change in demand.

→ supply/productivity

$P = \beta K$ → supply/productivity is derived from production function.

Capital stock

$$\frac{dP}{dt} = \beta \frac{dK}{dt}$$

→ change in capital over time is known as investment.

$$\frac{dK}{dt} = I$$

$$\frac{dP}{dt} = \beta I$$

In equilibrium we require $\boxed{S = I}$

where ⇒ $S = sY$

mps income.

∴ In equilibrium $\frac{dP}{dt} = \beta \cdot S = \beta \cdot sY$

∴ or $\frac{dY}{dt} = \beta \cdot sY$

or, $\frac{dY}{dt} - \beta s Y = 0$

const

$$Y(t) = A \cdot e^{-\int \beta s \cdot dt}$$

$$= A e^{-\beta \cdot s \cdot t + c}$$

$$= A e^{-\beta \cdot st} \cdot e^{c}$$

$$= \lambda e^{-\beta \cdot st}$$

At initial condition $t = 0$ ∴ $Y(t) = Y_0$

At initial condition $t=0$ $\therefore y(t) = y_0$

$y(0) = \lambda e^{-\beta \cdot s 0}$

$\lambda = y_0$

$\therefore \boxed{y(t) = y_0 e^{-\beta s t}}$

is solution.