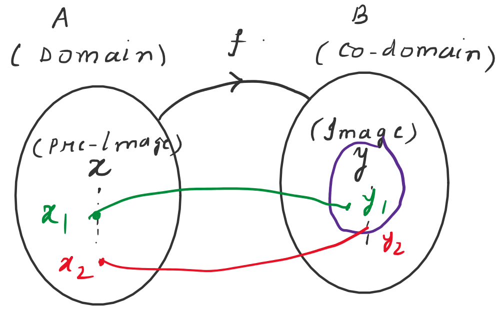
Functions



Eg: $y = f(x)$ $\Rightarrow$ Mapping b/w 2 set of no.s

Range of function: set of y-values that have a pre-image in the domain.

$\therefore$ Range $\subseteq$ co-domain.

Type of Functions:

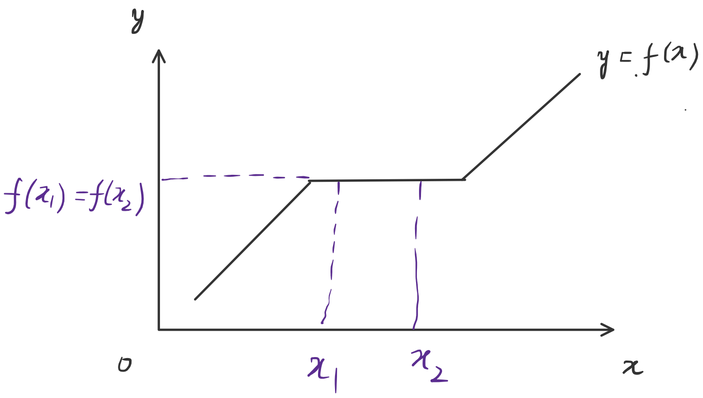
i) One-one function (Injective function)

If $x_1 \neq x_2 \in D_f$, then $f(x_1) \neq f(x_2)$, where $f(x_1), f(x_2) \in R_f$

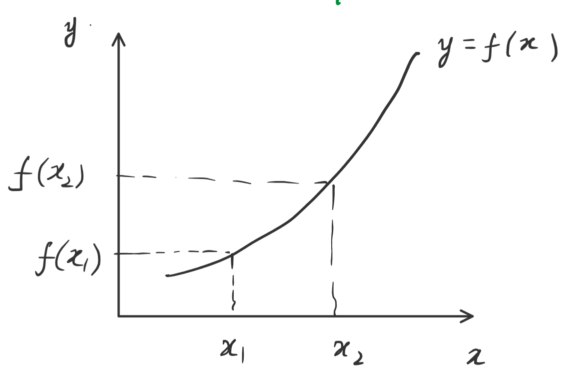
or, assume $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Note: Strictly monotone fn: $f' > 0$ / $f' < 0$.

Strictly monotone fn are one-one (increasing / decreasing).



Non-decreasing fn / Monotone increasing fn ($f' \geq 0$)



Strictly monotone increasing fn ($f' > 0$)

Q. $f(x) = x^3 - 3x^2 + 6x - 5$

Defn: $f(x_1) = f(x_2)$

$$x_1^3 - 3x_1^2 + 6x_1 - \cancel{5} = x_2^3 - 3x_2^2 + 6x_2 - \cancel{5}$$

$$(x_1^3 - x_2^3) - 3(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$$

$$(x_1-x_2)\left[x_1^2+x_1x_2+x_2^2-3(x_1+x_2)+6\right]=0$$

$$(x_1-x_2)\left[x_1^2+x_1x_2+x_2^2-3x_1-3x_2+6\right]=0$$

Either: $\boxed{x_1=x_2}$ or, $x_1^2+x_1x_2+x_2^2-3x_1-3x_2+6=0$.

↳ $f(x)$ is one-one.

Alt: $f(x)=x^3-3x^2+6x-5$

$f'(x)=\underset{>0}{3}x^2-6x+6$.

$D_{f'(x)}=-8<0$, $\Rightarrow f'(x)>0$.

↳ $f(x)$ is Monotone-increasing

$\Rightarrow f(x)$ is one-one.

ii) Onto Function [Surjective Function].

Co-domain = Range,

i.e every $y\in R_f$, has atleast one pre-image.

For $y=f(x)$, check if we have $x\in D_f$ for every $y\in R_f$.

Q. $f(x)=4x^3+7$; $f:\mathbb{R}\to\mathbb{R}$.

Let $y=4x^3+7$.

$y-7=4x^3$

$x=\left(\frac{y-7}{4}\right)^{1/3}$.

$(1)^{\frac{1}{3}}=1, \boxed{\omega,\ \omega^2}\to$ Imaginary

$(-27)^{1/3}=-3,-3\omega,-3\omega^2$

$\downarrow$

$\in\mathbb{R}$.

If $y-7\geq 0\Rightarrow x\geq 0\in D_f$.

$y-7<0\Rightarrow x<0\in D_f$

$\Rightarrow$ onto fn.

One-one: $f'=$

Q. Let $f: \mathbb{R} \to \mathbb{R}$ be defined as: $f(x) = \dfrac{e^{|x|} - e^{-x}}{e^{x} + e^{-x}}$. Then f is:

(a) One-one & Onto

(b) One-one & not onto

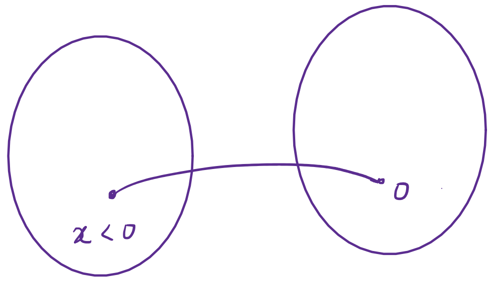
(c) Not one-one & onto

(d) Not one-one & not onto.

One-one:

$$x > 0 \Rightarrow f(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}.$$

$$x < 0 \Rightarrow f(x) = \frac{e^{-x} - e^{-x}}{e^{x} + e^{-x}} = 0.$$



$x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2) = 0$.

Note: For $f: \mathbb{R}_+ \to \mathbb{R} \Rightarrow$ one-one

For $f: \mathbb{R} \to \mathbb{R} \Rightarrow$ not one-one.

Onto: $x > 0 \Rightarrow f(x) = \dfrac{e^{2x} - 1}{\underbrace{e^{2x} + 1}_{>0}}$ $\Rightarrow \boxed{x = \frac{1}{2} \ln\left(\frac{1+y}{1-y}\right)}$

$y = 1 \Rightarrow x \notin D_f$.

Let $y = f(x) \Rightarrow y(e^{2x} + 1) = e^{2x} - 1$

$\Rightarrow e^{2x}(y - 1) + y + 1 = 0$.

$\therefore$ Not onto.

HW.

Q. Consider 2 sets A & B having 'm' & 'n' elements respectively. How many functions $f: A \to B$ are possible?