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Partial.

$d\,f(y,t) = M\,dy + N\,dt$

for exact differential equation $\frac{\partial M}{\partial t} = \frac{\partial N}{\partial y}$

$[M\,dy + N\,dt = 0]$

Q ⇒ $\underbrace{(6yt + 9y^2)}_{M}\,dy + \underbrace{(3y^2 + 8t)}_{N}\,dt = 0$

$\frac{\partial M}{\partial t} = 6y$ $\quad \frac{\partial N}{\partial y} = 6y$

$f(y,t) = \int (6yt + 9y^2)\,dy + z(t)$

$\frac{\partial f}{\partial t} = 3y^2 + z'(t)$ $\quad \to z'(t)$

in equation $N = 3y^2 + 8t$

$f(y,t) = \int (6yt + 9y^2)\,dy + z(t)$

$\frac{3}{2}t \cdot y^2 + \frac{3}{}9y^3 + z(t) + c$

$z'(t) = 8t$

$\frac{\partial z}{\partial t} = 8t$

$f(y,t) = \int (6y \ldots) \ldots + z(t) +$

$= \frac{\cancel{6}^{3} t \cdot y^2}{\cancel{2}} + \frac{\cancel{9}^{3} y^3}{\cancel{3}}$

$\boxed{f(y,t) = 3ty^2 + 3y^3 + 4t^2 + c}$

$-\frac{\partial z}{\partial t} = 8t$

$\int \partial z = 8 \int t \cdot dt$

$z(t) = \frac{\cancel{8}^{4} t^2}{\cancel{2}} = 4t^2$

$\underline{Q}$

$$\frac{dy}{dt} = y^2 t$$

$$\int \frac{dy}{y^2} = \int t \cdot dt$$

$$\int y^{-2} dy = \int t \cdot dt$$

$$\frac{y^{-2+1}}{-2+1} + c_1 = \frac{t^2}{2} + c_2$$

$$\frac{y^{-1}}{-1} + c_1 = \frac{t^2}{2} + c_2$$

$$-\frac{1}{y} + c_1 = \frac{t^2}{2} + c_2$$

$$-\frac{1}{y} = \frac{t^2}{2} + c_2 - c_1$$

$$-\frac{1}{y} = \frac{t^2 + 2c_2 - 2c_1}{2}$$

$$-y = \frac{2}{}$$

$$-y = \frac{2}{t^2+2c_2-2c_1}$$

$$y = \frac{-2}{t^2+2c_2-2c_1}$$

Let $c = 2c_2 - 2c_1$

$$\boxed{y = \frac{-2}{t^2+c}}$$

(3) Given that

$$t^2\,dy + y^3\,dt = 0$$

$$t^2\,dy = -y^3\,dt$$

$$\frac{dy}{y^3} = -\frac{dt}{t^2}$$

$$\int y^{-3}\,dy = -\int t^{-2}\,dt$$

$$\frac{y^{-3+1}}{-3+1} + c_1 = -\frac{t^{-2+1}}{-2+1} + c_2$$

$$\frac{y^{-2}}{-2} + c_1 = -\frac{t^{-1}}{-1} + c_2$$

$$\frac{y^{-2}}{-2} = t^{-1} + c_2 - c_1$$

$$y^{-2} = -2t^{-1} + 2(c_2-c_1)$$

$$\frac{1}{y^2} = \frac{-2}{t} + 2(c_2-c_1)$$

$$\frac{1}{y^2} = \frac{-2}{t} + 2(c_2 - c_1)$$

$$\frac{1}{y^2} + \frac{2}{t} - 2(c_2 - c_1) = 0$$

ans $$\boxed{f(y,t) = \frac{1}{y^2} + \frac{2}{t} - c = 0}$$

Application :

$Q^d = c + bP$

$Q^s = g + hP$

rate of change of price with time is related to the excess demand.

Find the time path of price and also comment on it's stability.

$$\frac{dP}{dt} \propto (Q^d - Q^s)$$

$$\frac{dP}{dt} = m(Q^d - Q^s)$$

$$\frac{dP}{dt} = m\left((c + bP) - (g + hP)\right)$$

$$\frac{dP}{dt} = m\left[c + bP - g - hP\right]$$

$$\frac{dP}{dt} \quad \ldots \quad \left[P(b-h) + (c-g)\right]$$

$\frac{d}{dt}$

$$\frac{dP}{dt} = m\left[P(b-h) + (c-g)\right]$$

$$\frac{dP}{dt} = P\,m(b-h) + m(c-g)$$

$$\frac{dP}{dt} - m(b-h)\,P^{1} = m(c-g)$$

This is first order, first degree non-homogenous differential eqn.

PI: Price over time is const i.e. $P = \bar{P}$

i.e. $\frac{dP}{dt} = 0$. $0 - m(b-h)\,\bar{P} = (c-g)m$

$\therefore \bar{P} = \frac{m(c-g)}{m(b-h)}$

CF: $\frac{dP}{dt} - m(b-h)\,P(t) = 0$ $P_p = \boxed{\bar{P} = \frac{c-g}{(h-b)}}$

equilibrium.

$$P(t) = Ae^{-\int g\,dt}$$

$$= Ae^{+\int (mb-mh)\,dt}$$

$$= Ae^{(mb-mh).t + C.}$$

$$= Ae^{m(b-h)t} \cdot e^{C}$$

$$= \lambda e^{m(b-h)t}$$

P

$P_c = \lambda e^{m(b-h)t}$

Timepar

$\therefore P(t) = P_p + P_c = \dfrac{c-g}{h-b} + \lambda e^{m(b-h)t}$

at the initial condition $t = 0$ $\Rightarrow P(0) = \dfrac{c-g}{h-b} + \lambda e^{m(b-h)\times 0}$

$$P(0) = \frac{c-g}{h-b} + \lambda$$

$$\lambda = P(0) - \left(\frac{c-g}{h-b}\right)$$

$\therefore$ Required soln is

$$P(t) = \frac{c-g}{h-b} + \left[P(0) - \left(\frac{c-g}{h-b}\right)\right] e^{m(b-h)t}$$

ans

if point elasticity is -1, find $D(Q)$?

$$\epsilon = \frac{dQ}{dP} \times \frac{P}{Q} = -1$$

$$\frac{dQ}{dP} = -\frac{Q}{P}$$

$$\int \frac{dQ}{Q} = -\int \frac{dP}{P}$$

$$\int \frac{dQ}{Q} = -\int \frac{dP}{P}$$

$$\log Q + c_1 = -\log P + c_2$$

$$\log Q + \log P = c_2 - c_1$$

$$\log(QP) = \log c$$

$$QP = c$$

$$Q = \frac{c}{P}$$

demand function