09 February 2024 13:47

Conversion between Cylindrical and Cartesian Coordinates

The rectangular coordinates (x, y, z) and the cylindrical coordinates (r, θ, z) of a point are related as follows:

These equations are used to convert from cylindrical coordinates to rectangular coordinates.

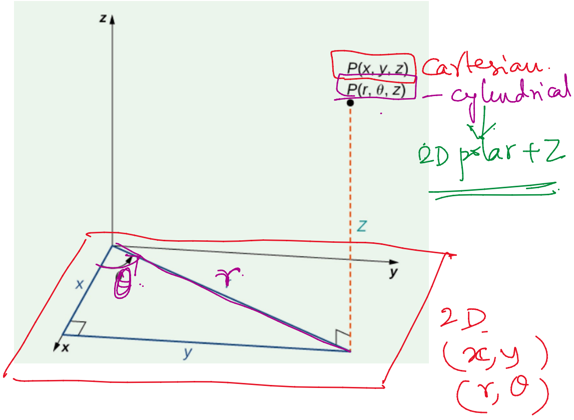
- $x = r\cos\theta$
- $y = r\sin\theta$
- $z = z$

|| 2D polar form.

These equations are used to convert from rectangular coordinates to cylindrical coordinates

1. $r^2 = x^2 + y^2$
2. $\tan\theta = \frac{y}{x}$
3. $z = z$

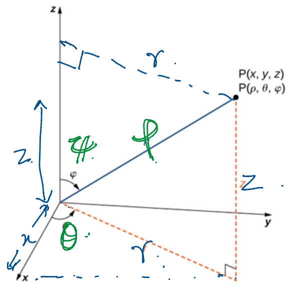
|| 2D polar form.



Definition: spherical coordinate system

In the *spherical coordinate system*, a point P in space (Figure 11.6.9) is represented by the ordered triple (ρ, θ, φ) where

- ρ (the Greek letter rho) is the distance between P and the origin $(\rho \neq 0)$;
- θ is the same angle used to describe the location in cylindrical coordinates;
- φ (the Greek letter phi) is the angle formed by the positive z-axis and line segment $\overline{OP}$, where O is the origin and $0 \leq \varphi \leq \pi$.



$\rho^2 = r^2 + z^2$

$\rho^2 = x^2 + y^2 + z^2$

$\tan\theta = \frac{y}{x}$

$\cos\varphi = \frac{z}{\rho}$

HOWTO: Converting among Spherical, Cylindrical, and Rectangular Coordinates

Rectangular coordinates (x, y, z), cylindrical coordinates (r, θ, z), and spherical coordinates (ρ, θ, φ) of a point are related as follows:

Convert from spherical coordinates to rectangular coordinates x, y, z needed.

These equations are used to convert from spherical coordinates to rectangular coordinates.

- $x = \boxed{\rho\sin\varphi}\cos\theta$
- $y = \boxed{\rho\sin\varphi}\sin\theta$
- $z = \rho\cos\varphi$

$x = r\cos\theta$ $\quad r = \rho\sin\varphi$

$y = r\sin\theta$

$z = \rho\cos\varphi$

Convert from rectangular coordinates to spherical coordinates ρ, θ, φ needed.

These equations are used to convert from rectangular coordinates to spherical coordinates.

- $\rho^2 = x^2 + y^2 + z^2$
- $\tan\theta = \frac{y}{x}$
- $\varphi = \arccos\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$.

$\cos\varphi = \frac{z}{\rho} = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$x = r\cos\theta$ $\quad \rho\cos\varphi = z$

$\boxed{x = \rho\sin\varphi\cos\theta}$ $\quad \sin\varphi = \frac{r}{\rho}$

$\boxed{r = \rho\sin\varphi}$

$y = x\tan\theta$

$y = \rho\sin\varphi\cos\theta \cdot \frac{\sin\theta}{\cos\theta} = \rho\sin\varphi\sin\theta$

$\boxed{y = \rho\sin\varphi\sin\theta}$ $\quad \boxed{z = \rho\cos\varphi}$

Convert from spherical coordinates to cylindrical coordinates

These equations are used to convert from spherical coordinates to cylindrical coordinates.

- $r = \rho\sin\varphi$
- $\theta = \theta$
- $z = \rho\cos\varphi$

Spherical (ρ, θ, φ)

cylindrical (r, θ, z)

Convert from cylindrical coordinates to spherical coordinates

These equations are used to convert from cylindrical coordinates to spherical coordinates.

- $\rho = \sqrt{r^2 + z^2}$
- $\theta = \theta$
- $\varphi = \arccos\left(\frac{z}{\sqrt{r^2 + z^2}}\right)$

$\rho^2 = r^2 + z^2$

$\cos\varphi = \frac{z}{\rho} = \frac{z}{\sqrt{r^2+z^2}}$

Describe the surfaces with the given spherical equations.

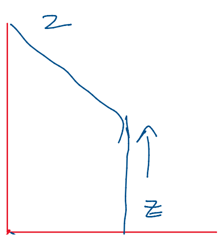
a. $\theta = \frac{\pi}{3}$

b. $\varphi = \frac{5\pi}{6}$

ρ, θ, φ

$\downarrow$

r, θ, z.



$r \geq 0$

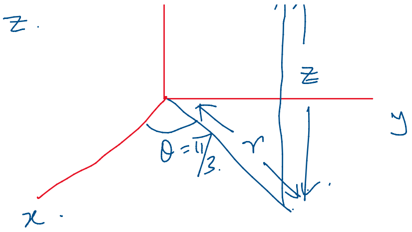
b. $\varphi = \frac{5\pi}{6}$

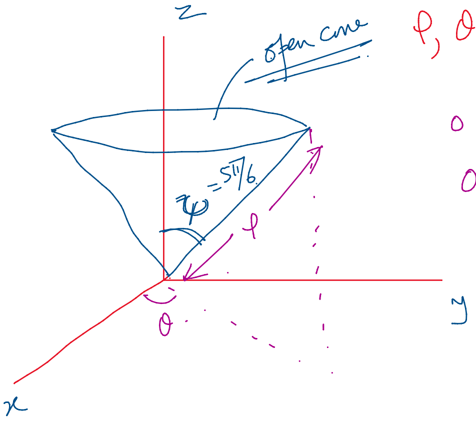
c. $\rho = 6$

d. $\rho = \sin\theta \sin\varphi$

$\rho^2 = x^2 + y^2 + z^2$

$r, \theta, z.$





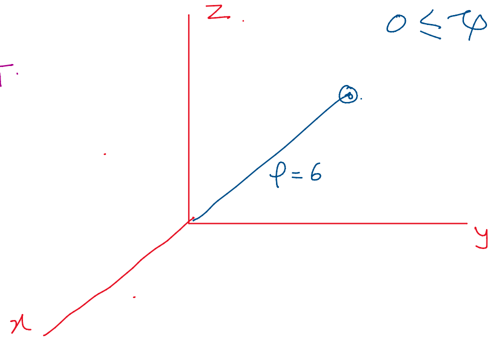
ρ, θ, φ

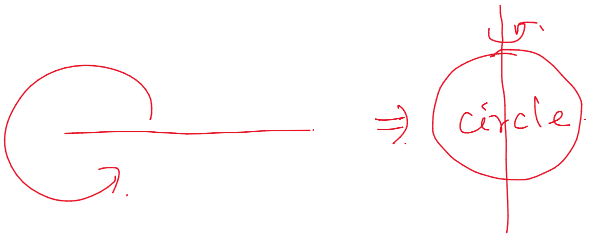
$0 \le \rho < \infty.$

$0 \le \theta \le 2\pi.$

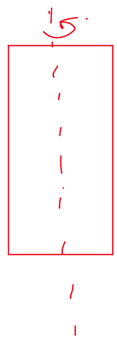
$0 \le \theta \le 2\pi.$

$0 \le \varphi \le 2\pi.$





$\Rightarrow$ sphere.



$\Rightarrow$ cylinder.

- $r = \rho \sin\varphi$
- $\theta = \theta$
- $z = \rho \cos\varphi$

$\rho = \sin\theta \sin\varphi$

$\rho^2 = \rho \sin\theta \sin\varphi = (\rho \sin\varphi)\sin\theta = r\sin\theta = y.$

$\rho^2 = y$

$x^2 + y^2 + z^2 = y.$

$x^2 + y^2 - y + z^2 = 0.$

$x^2 + \left[y^2 - 2\cdot\frac{1}{2}y + \left(\frac{1}{2}\right)^2\right] - \left(\frac{1}{2}\right)^2 + z^2 = 0.$

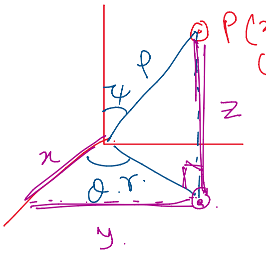
$x^2 + \left(y - \frac{1}{2}\right)^2 + z^2 = \left(\frac{1}{2}\right)^2.$ $\Rightarrow$ sphere with center $(0, \frac{1}{2}, 0)$ and radius $= \frac{1}{2}$.

Eqn of a sphere

$(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 = \rho^2$ where the center is (α, β, γ)

Convert the rectangular coordinates $(-1, 1, \sqrt{6})$ to both spherical and cylindrical coordinates.

II



$x = -1 \quad y = 1 \quad z = \sqrt{6}$.

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$.

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(-1) = \frac{3\pi}{4}$.

$r = \sqrt{x^2+y^2} = \sqrt{2}$.

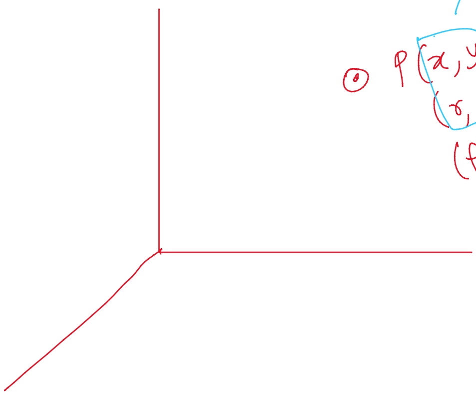
$\rho^2 = r^2 + z^2 = 8$

$\rho = 2\sqrt{2}$.

$\cos\varphi = \frac{z}{\rho} = \frac{\sqrt{6}}{2\sqrt{2}} = \frac{\sqrt{3}}{2}$

$\varphi = \frac{\pi}{6}$.

2D $\rightarrow$ $\boxed{x = r\cos\theta \quad y = r\sin\theta \quad r^2 = x^2+y^2}$



P (x, y, z)

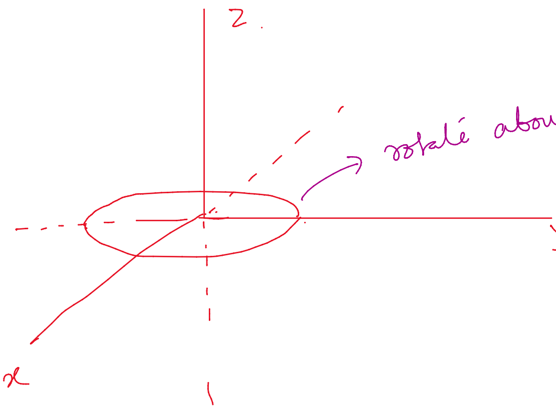
(r, θ, z)

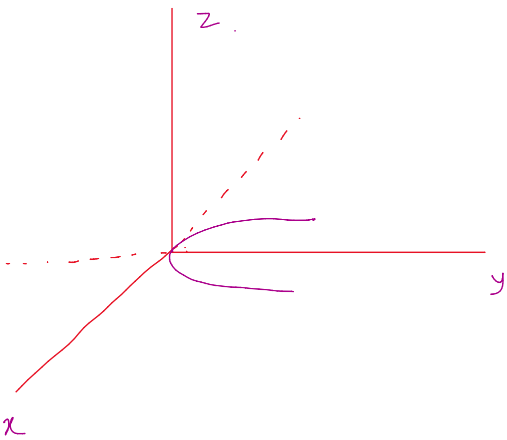
(ρ, θ, φ)

Cartesian $\Longleftrightarrow$ cylindrical

$\rho^2 = x^2+y^2+z^2 = r^2+z^2$.

$\varphi = \cos^{-1}\frac{z}{\rho}$.





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Conicoids