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Test for Heteroscedasticity

① Glejer Test :

$$|\hat{u}_i| = \beta_1 + \beta_2 X_i + v_i$$

$$|\hat{u}_i| = \beta_1 + \beta_2 \sqrt{X_i} + v_i$$

$$|\hat{u}_i| = \beta_1 + \beta_2 \frac{1}{X_i} + v_i$$

$$|\hat{u}_i| = \beta_1 + \beta_2 \frac{1}{\sqrt{X_i}} + v_i$$

$$|\hat{v}_i| = \sqrt{\beta_1 + \beta_2 X_i} + v_i$$

$$|\hat{u}_i| = \sqrt{\beta_1 + \beta_2 X_i^2} + v_i$$

Non-linear.

regression

$E(v_i)$, $E(v_i)$ } 0

$E(x_i v_j) = 0$

② Goldfeld-Quandt Test : if one assume that the heteroscedastic variance, σ_i^2 is positively related to one of the explanatory variables in regression model $Y_i = \beta_1 + \beta_2 X_i + u_i$ and σ_i^2 is positively related to X_i as

$$\sigma_i^2 = \sigma^2 X_i^2$$

① Order or rank the observation according to the value of X_i, beginning with lowest X value.

② Omit 'c' central observations, where c is specified a priori and divide the ~~remaining~~ remaining $(n-c)$ observations in two groups each of $\frac{n-c}{2}$ observations.

(3) Fit separate OLS regressions to the first $\frac{(n-c)}{2}$ observations and the last $\frac{(n-c)}{2}$ observations and obtain the respective residual sum of squares RSS_1 and RSS_2. RSS_1 representing the RSS from the regression corresponding to the smaller x_i values and RSS_2 that from the larger x_i values. These RSS each have $\frac{(n-c)}{2} - k$ or $\left(\frac{n-c-2k}{2}\right)$ degree of freedom, where k is the no. of parameters to be estimated.

(4) Compute the ratio $\lambda = \frac{RSS_2/df}{RSS_1/df}$

If we assume u_i are normally distributed and if the assumption of homoscedasticity is valid then it can be shown that λ follows F-distribution with each numerator and denominator df $\frac{(n-c-2k)}{2}$.

If computed $\lambda (=F)$ is greater than critical F at α level of significance then H_0 is rejected & heteroscedasticity is likely present.

Functional forms of Regression

(1) Double-log, Log linear or Constant elasticity model

Eg: $Y_i = \alpha_0 X_i^{-\beta} e^{u_i}$

$$\ln Y_i = \ln \alpha_0 - \beta \ln X_i + u_i$$

$\ln Y_i = \ln \alpha_0 + \beta \ln X_i + u_i$

$Y_i^* = \alpha_0^* - \beta X_i^* + u_i$

↳ elasticity coeff between x and y

② Semi-log Model:

(a) $\ln Y_i = \alpha_0 + \alpha_1 X_i + u_i$

and (b) $Y_i = \beta_0 + \beta_1 \ln X_i + u_i$

(a) $\alpha_1 = \frac{d}{dx_i}(\ln Y_i) = \frac{1}{Y_i} \cdot \frac{dY_i}{dx_i} = \frac{dY/Y}{dX} = \frac{\Delta Y/Y}{\Delta X}$

$\alpha_1 = \frac{\text{prop change in } Y}{\text{absolute change in } X}$

b) $\beta_1 = \frac{dY_i}{d\log X_i} = \frac{\Delta Y}{\Delta X/X} = \frac{\text{absolute chng in } Y}{\text{proportional change in } X}$

③ Reciprocal Transformation or Hyperbolic Models

(a) $Y_i = \alpha + \beta\left(\frac{1}{X_i}\right) + u_i$

If α and β are positive, this model shows that Y decreases non linearly as X increases

(b) Another form of hyperbolic model is log-hyperbolic model which is of form $\ln Y_i = \alpha - \beta\left(\frac{1}{X_i}\right) + u_i$

or, $Y_i = e^{\alpha - \beta/X_i}$

Name of function	Relation	Slope
1. Linear	(i) $Y = \alpha + \beta X$	$+\beta$
	(ii) $Y = \alpha - \beta X$	$-\beta$
2. Quadratic	(i) $Y = \alpha + \beta_1 X - \beta_2 X^2$	$(+)\ (\beta_1 - 2\beta_2 X)$
	(ii) $Y = \alpha - \beta_1 X - \beta_2 X^2$	$(-)\ (\beta_1 - 2\beta_2 X)$
3. Hyperbolic	(i) $Y = \alpha + \beta / X$	$-\ \beta / X^2$
	(ii) $Y = \alpha - \beta / X$	$(+)\ \beta / X^2$
4. Semilog	(i) $\log_e Y = \alpha + \beta X$ or $Y = e^{\alpha + \beta X}$	$(+)\ \beta Y$
	(ii) $\log_e Y = \alpha - \beta X$ or $Y = e^{\alpha - \beta X}$	$(-)\ \beta Y$
	(iii) $Y = \alpha + \beta \log_e X$ or $e^Y = \alpha + X^{\beta}$	$(+)\ \beta / X$
	(iv) $Y = \alpha - \beta \log_e X$ or, $e^Y = \alpha + X^{-\beta}$	$(-)\ \beta / X$
5. Log-quadratic	(i) $\log_e Y = \alpha + \beta_1 X - \beta_2 X^2$	$(+)\ Y(\beta_1 - 2\beta_2 X)$
	(ii) $\log_e Y = \alpha - \beta_1 X - \beta_2 X^2$ ✓	$(-)\ Y(\beta_1 - 2\beta_2 X)$
6. Log-inverse	(i) $\log_e Y = \alpha + \frac{\beta}{X}$ or $Y = e^{\alpha + \beta / X}$	$-\ \frac{\beta}{X^2} \cdot Y$
	(ii) $\log_e Y = \alpha - \beta / X$ or $Y = e^{\alpha - \beta / X}$	$+\ \beta / X^2 \cdot Y$
7. Double-log	(i) $\log_e Y = \alpha + \beta \log_e X$ or $Y = \alpha X^{\beta}$	$+\ \frac{\beta Y}{X}$
	(ii) $\log_e Y = \alpha - \beta \log_e X$ or $Y = \alpha X^{-\beta}$	$-\ \frac{\beta Y}{X}$

$\frac{dY}{dX}$

$e^{\alpha + \beta X}$

$Y\beta$