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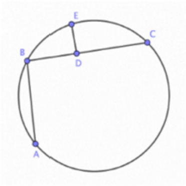
In the figure below, E is the midpoint of the arc $ABEC$ and the segment ED is perpendicular to the chord BC at D. If the length of the chord AB is l_1, and that of the segment BD is l_2, determine the length of DC in terms of l_1, l_2.



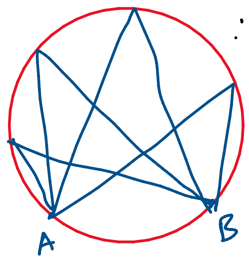
$AE = EC$.

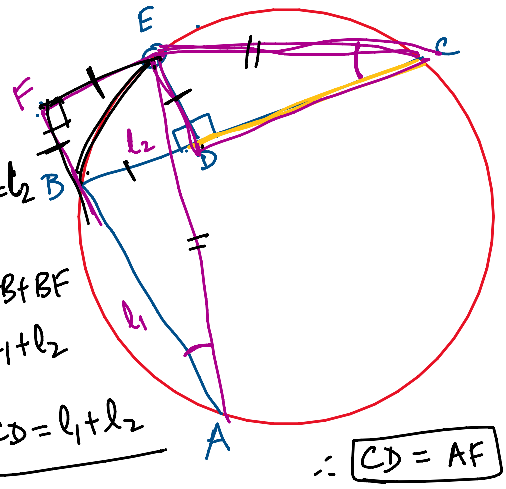
ΔECD is rotated around E such that EC coincides with AE and $\angle ECD$ coincides with $\angle EAB$.

$\therefore \Delta EAF \cong \Delta ECD$

EDBF is a square.

$\therefore BF = BD = l_2$





$AF = AB + BF$
$= l_1 + l_2$

$\therefore CD = l_1 + l_2$

$\therefore \boxed{CD = AF}$

Let A, B and C be three points on a circle of radius 1.

(a) Show that the area of the triangle ABC equals $\frac{1}{2}(\sin(2\angle ABC) + \sin(2\angle BCA) + \sin(2\angle CAB))$

(b) Suppose that the magnitude of $\angle ABC$ is fixed. Then show that the area of the triangle ABC is maximized when $\angle BCA = \angle CAB$

(c) Hence or otherwise, show that the area of the triangle ABC is maximum when the triangle is equilateral.

area of $\Delta ABC = \frac{1}{2}\left[\sin 2\angle ABC + \sin 2\angle BCA + \sin 2\angle CAB\right]$

$\angle BOC = 2\angle A \quad \angle AOB = 2\angle C \quad \angle AOC = 2\angle B$.

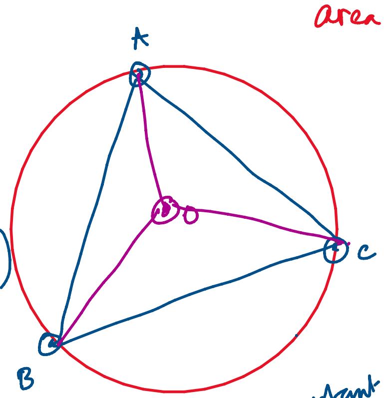
$OA = OB = OC = 1$

area of $\Delta ABC = \Delta AOB + \Delta BOC + \Delta AOC$.

$\Delta AOB = \frac{1}{2} \cdot AO \cdot BO \cdot \sin\angle AOB = \frac{1}{2}\sin 2\angle C$.

$\Delta BOC = \frac{1}{2}\sin 2\angle A \quad \Delta AOC = \frac{1}{2}\sin 2\angle B$.

area of $\Delta ABC = \frac{1}{2}\left[\sin 2A + \sin 2B + \sin 2C\right]$



$\sin A + \sin B$
$= 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$A + B + C = 180^\circ$

$A + C = 180 - B = \text{const}$

$\Delta = \frac{1}{2}\left[\sin 2A + \underbrace{\sin 2B}_{\text{constant}} + \sin 2C\right]$

$\Delta_{max} \rightarrow (\sin 2A + \sin 2C)_{max}$.

$Z = \sin 2A + \sin 2C$
$= 2\sin(A+C)\cos(A-C)$

$\Delta_{max} \rightarrow (\sin 2A + \sin 2C)_{max}$. $= 2\boxed{\sin(A+C)}\boxed{\cos(A-C)}$

$\sin(A+C)$: const.

$\cos(A-C) = 1$

$\Rightarrow \boxed{A = C}$

$\Delta = \frac{1}{2}\left[\sin 2A + \sin 2B + \sin 2C\right]$

$= \frac{1}{2}\left[2\sin(A+B)\cos(A-B) + 2\sin C\cos C\right]$ $A+B = 180 - C$

$\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$

$= \frac{1}{2}\left[2\sin(180-C)\cos(A-B) + 2\sin C\cos\{180-(A+B)\}\right]$

$= \sin C\cos(A-B) + \sin C\{-\cos(A+B)\} = \sin C\left[\cos(A-B) - \cos(A+B)\right]$

$= \sin C \cdot 2\sin A\sin B . = 2\sin A\sin B\sin C.$ $0 < A, B, C < 180^\circ$

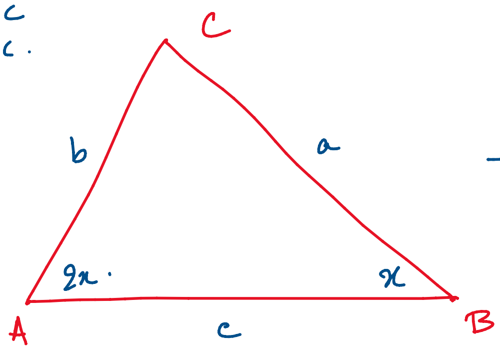
Δ_{max} when $\sin A = \sin B = \sin C$ AM $\geq$ GM $\frac{x+y+z}{3} \geq \sqrt[3]{xyz}$

$A = B = C$

In a triangle ABC, angle A is twice the angle B. Then which of the following has to be true?

(A) $a^2 = b(b+c)$
(B) $b^2 = a(a+c)$
(C) $c^2 = a(a+b)$
(D) $ab = c(a+c)$

$a^2 = b^2 + bc$
$b^2 = a^2 + ac$.



cosine rule

$a^2 = b^2 + c^2 - 2bc\cos 2x$ . .

$-\left(b^2 = a^2 + c^2 - 2ac\cos x\right)$

$a^2 - b^2 = b^2 - a^2 + 2ac\cos x - 2bc\cos 2x$

$2(b^2 - a^2) + 2c\left[a\cos x - b\cos 2x\right] = 0$

$(b^2 - a^2) + c(a\cos x - b\cos 2x) = 0$ —(1)

$\cos 2x = 2\cos^2 x - 1$

$= 2\frac{a^2}{4b^2} - 1$

$= \frac{a^2}{2b^2} - 1$

$= \frac{a^2 - 2b^2}{2b^2}$

$b\cos 2x = \frac{a^2 - 2b^2}{2b}$.

sin rule $\frac{a}{\sin 2x} = \frac{b}{\sin x}$.

$\frac{a}{b} = \frac{\sin 2x}{\sin x} = \frac{2\cancel{\sin x}\cos x}{\cancel{\sin x}}$

$\boxed{\cos x = \frac{a}{2b}}$ —(2)

$a\cos x = \frac{a^2}{2b}$.

from (1)

$b^2 - a^2 + c\left[\frac{a^2}{2b} - \frac{(a^2 - 2b^2)}{2b}\right] = 0.$

$b^2 - a^2 + c\left[\frac{2b^2}{2b}\right] = 0$ $b^2 - a^2 + bc = 0.$

$a^2 = b(b+c)$

Consider a triangle ABC with the sides a, b, c in A.P. Then the largest possible value of the angle B is

(A) 60°
(B) $67\frac{1}{2}^\circ$;
(C) 75°
(D) $82\frac{1}{2}^\circ$.

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$= \frac{3b^2 - 2ac}{2ac}$

$= \frac{3}{2}\left(\frac{b^2}{ac}\right) - 1$

$(\cos B) = \frac{3}{2} - 1 = \frac{1}{2} = \cos 60^\circ$.

$a + c = 2b$

$a^2 + c^2 + 2ac = 4b^2$

$a^2 + c^2 - b^2 = 3b^2 - 2ac$.

AM $\geq$ GM

$a + c \geq 2\sqrt{ac}$.

$2b \geq 2\sqrt{ac}$.

If B is max

$\cos B$ is minimum.

$\Rightarrow \frac{b^2}{ac}$ is minimum

$= \frac{3}{2}\left(\frac{b^2}{ac}\right) - 1$

$\Rightarrow \frac{b^2}{ac}$ is minimum

$\left(\frac{b^2}{ac}\right)_{min} = 1$

$(\cos B)_{min} = \frac{3}{2} - 1 = \frac{1}{2} = \cos 60^\circ$.

$B_{max} = 60^\circ$

$a + c \geq 2\sqrt{ac}$.

$2b \geq 2\sqrt{ac}$.

$b \geq \sqrt{ac}$

$b^2 \geq ac$.

$\frac{b^2}{ac} \geq 1$

Consider a circle with centre O. Two chords AB and CD extended intersect at a point P outside the circle. If $\angle AOC = 43^\circ$ and $\angle BPD = 18^\circ$, then the value of $\angle BOD$ is

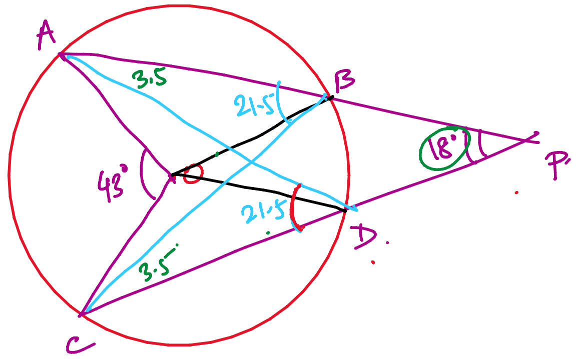
(A) 36°

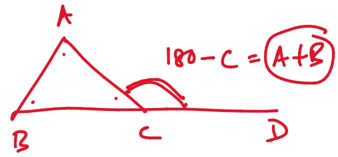
(B) 29°

✓ (C) 7°

(D) 25°,

$\angle BOD = 2\angle BCD = 7^\circ$





Consider a triangle ABC. The median AD meets the side BC at the point D. A point E on AD is such that $AE : DE = 1 : 3$. The straight line BE extended meets the side AC at a point F. Then $AF : FC$ equals

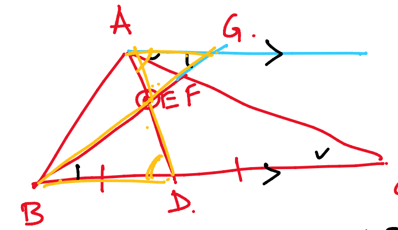
(A) $1 : 6$;

(B) $1 : 7$;

(C) $1 : 4$;

(D) $1 : 3$.

4 9.



$\frac{AE}{DE} = \frac{1}{3}$

$\Delta AFG \sim \Delta CFB.$

$\frac{AF}{FC} = \frac{AG}{BC}$ — (1)

$\Delta AEG \sim \Delta DEB.$

$\frac{AF}{FC} = \frac{1}{6}$

$BD = \frac{1}{2} BC$.

$\frac{AG}{BD} = \frac{AE}{ED} = \frac{1}{3}.$

$\frac{AG}{BC} = \frac{1}{6}.$

$\frac{AG}{\frac{BC}{2}} = \frac{1}{3}$

$2\frac{AG}{BC} = \frac{1}{3}$

Let C be the circle $x^2 + y^2 + 4x + 6y + 9 = 0$. The point $(-1, -2)$ is

(A) inside C but not the centre of C;

(B) outside C;

(C) on C :

(D) the centre of C.

Suppose $ABCD$ is a quadrilateral such that $\angle BAC = 50°$, $\angle CAD = 60°$ $\angle CBD = 30°$ and $\angle BDC = 25°$. If E is the point of intersection of AC and BD, then the value of $\angle AEB$ is

(A) $75°$;
(B) $85°$;
(C) $95°$;
(D) $110°$.

The angles of a convex pentagon are in A.P. Then, the minimum possible value of the smallest angle is

(A) $30°$
(B) $36°$
(C) $45°$;
(D) $54°$.

A triangle ABC has a fixed base BC. If $AB : AC = 1 : 2$, then the locus of the vertex A is

(A) a circle whose centre is the midpoint of BC;
(B) a circle whose centre is on the line BC but not the midpoint of BC;
(C) a straight line;
(D) none of the above.

Two circles touch each other at P. The two common tangents to the circles, none of which pass through P meet at E. They touch the larger circle at C and D The larger circle has radius 3 units and CE has length 4 units. Then the radius of the smaller circle is
(A) 1 ;
(B) $\frac{5}{7}$;
(C) $\frac{3}{4}$;
(D) $\frac{1}{2}$.

Let ABC be a right angled triangle with $AB > BC > CA$. Construct three equilateral triangles BCP, CQA and ARB, so that A and P are on opposite sides of BC : B and Q are on opposite sides of CA; C and R are on opposite sides of AB. Then

(A) $CR > AP > BQ$

(B) $CR < AP < BQ$

(C) $CR = AP = BQ$

(D) $CR^2 = AP^2 + BQ^2$.

A building with ten storeys, each storey of height 3 metres, stands on one side of a wide street. From a point on the other side of the street directly opposite to the building, it is observed that the three uppermost storeys together subtend an angle equal to that subtended by the two lowest storeys. The width of the street is

(A) $6\sqrt{35}$ metres;
(B) $6\sqrt{70}$ metres;
(C) 6 metres;
(D) $6\sqrt{3}$ metres.