## Intervals

$( \ ) \to$ open interval.

$[ \ ] \to$ closed interval.

$\{ \ \} \to$ finite set.

Eg: $(a, b)$ : open interval $:= \{x \in \mathbb{R} : a < x < b\}$.

$[a, b]$ : closed interval $= \{x \in \mathbb{R} : a \leq x \leq b\}$.

$\to$ Bounded sets.

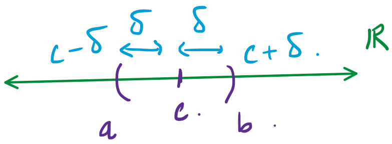
$(a, b]$ , $[a, b) \to$ Bounded sets.

Any interval whose both the end points are finite $\in \mathbb{R}$, it will be a bounded set.

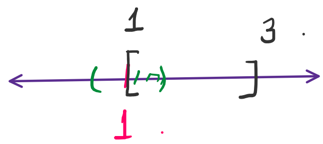
## Neighbourhood of a Point:

Consider $c \in \mathbb{R}$. A subset $S \subset \mathbb{R}$ is said to be a neighbourhood of $c$ if $\exists$ an open interval $(a, b)$ s.t. $c \in (a, b)$.

Consider $c \in \mathbb{R}$ and $\delta > 0$. Then the open interval $(c-\delta, c+\delta)$ is the $\delta$-Neighbourhood of pt '$c$'.




Eg: Consider $N = [1, 3]$. N is a neighbourhood of 1 ?

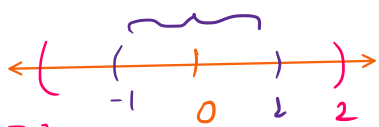



Note: A closed interval may not be a neighbourhood of pt $c \in \mathbb{R}$.

(i) Suppose we consider 2 neighbourhood $N_1$ and $N_2$ of pt $c \in \mathbb{R}$. Then $N_1 \cap N_2$ will also be a neighbourhood of pt '$c$'.

Eg: $c = 0$ , $N_1 = (-2, 2)$ $N_2 = (-1, 1)$.

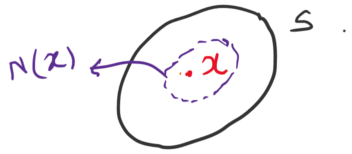
$N_1 \cap N_2 = (-1, 1)$ is also a neighbourhood of '$c$'




## Interior point:

Let $S \subset \mathbb{R}$. A point $x \in S$ is said to be an interior point of $S$ if $\exists$ a neighbourhood $N(x)$ of $x$ s.t. $N(x) \subseteq S$.

Let $S \subset \mathbb{R}$. A point $x \in S$ is said to be an interior point of $S$ if $\exists$ a neighbourhood $N(x)$ of $x$ s.t $\boxed{N(x) \subset S}$.




## Open Set:

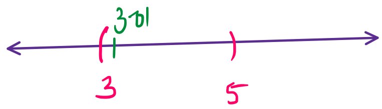
Suppose $S \subset \mathbb{R}$. Set $S$ will be an open set if each point of $S$ is an interior point.

Eg: $S = \{1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots\} \to \{\frac{1}{n}\}$

Consider pt $c = 1$. Neighbourhood of $1 \not\subset S$.

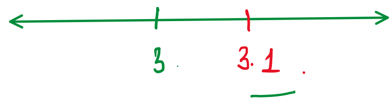
$S$ is not an open set.

Eg: $S = (3, 5) \Rightarrow$ open set.




$x \in S \Rightarrow 3 < x < 5$

$x > 3$ or $x < 5$




$\delta = 0.1$. 3 to 3.2 $\Rightarrow$ N

$\delta = 0.01 / 0.05 < 0.1$

Eg: $S = [3, 5] \Rightarrow$ not open set.

## Summary of Open sets:

| # / Op. | Finite | Infinite |
|---|---|---|
| Union | Open set | Open set |
| Int. | Open set | Not necessarily be open. |