power set is set of all subsets

No. of ~~the~~ elements in a power set is 2^n

ex if there are 4 element in set A

$$A = \{1, 3, 7, 9\}$$

then No. of subsets $= 2^4 = 16$

Power set:

Subsets of A: $\{\phi, \{1\}, \{2\}, \{3\}, \{7\}, \{9\}, \{1, 3\}, \{1, 7\} \ldots \{1, 3, 7, 9\}\}$

② Proper and Improper set.

Let $S = \{1, 2, 3, \ldots, 9 \ldots\}$

$n(S) = C_0$ ✓

$N = \{1, 4, 9, \ldots, 25, \ldots 100, \ldots\}$

$n(N) = C_1$ ✓

Now if $C_0 = C_1$ ie $n(S) = n(N)$

Now if $c_0 = c_1$ ie $n(S) = n(N)$

then $N \subseteq S$ (improper Subset)

but if $c_0 > c_1$ ie $n(S) > n(N)$

then $N \subset S$ (proper subset)

What / Given an example of improper subset.

Let $A = \{1, 4, 9, 16, 25, \dots, 100\}$

$B = \{1, 2, 3, 4, 5, 6, \dots, 10\}$

$n(A) = 10$
$n(B) = 10$

$\therefore A \subseteq B$

This proves Galileo's theorem which states that for every natural number, there is a corresponding square number so the cardinality of two sets are equal.

Functions / Mapping

Let U is the universal set

$X, Y \subseteq U$

$f : X \to Y$

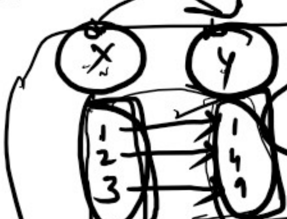
domain

let $X = \{1, 2, 3\}$

$Y = \{1, 4, 9\}$.

range

$Y = x^2 = f(x)$



$f(x)$

image of x

it means for every element of x

(f).

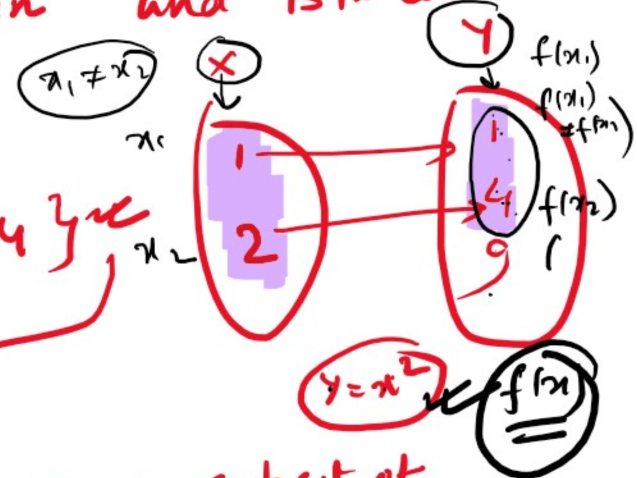
↳ f is mapping it means for every element of x (domain) there is an element y (codomain/range).

→ $f(x)$ is called image set

Ex: $A = \{1, 2\}$ A is domain and B is codomain

$f = f(a) = a^2$

$f(A) = \{1^2, 2^2\} = \{1, 4\}$



Now suppose $B = \{1, 4, 5\}$

$\therefore f(A)$ is subset of set B

So we can conclude image set is a subset of codomain.

\# If $f(x) = y$ ⇒ onto mapping

\# image set of x is the range

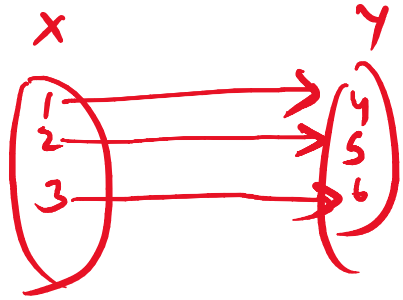
\# If $\forall\ x \in X \mid f(x) = y$, then f is onto mapping

\# If $\forall\ x_1 \neq x_2 \mid f(x_1) \neq f(x_2)$, then f is called one-to-one mapping.

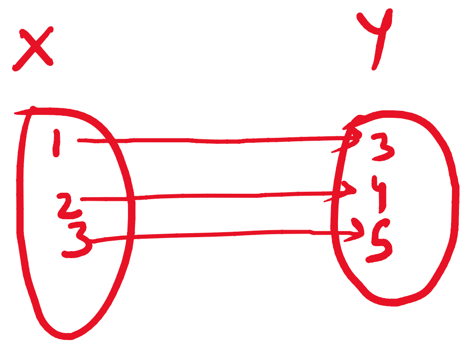
\# when we get elements of X from element of Y, in the inverse way, it is called correspondence and happens only when both onto and one-to-one mapping applies.

correspondence and mapping
onto and one-to-one mapping applies.

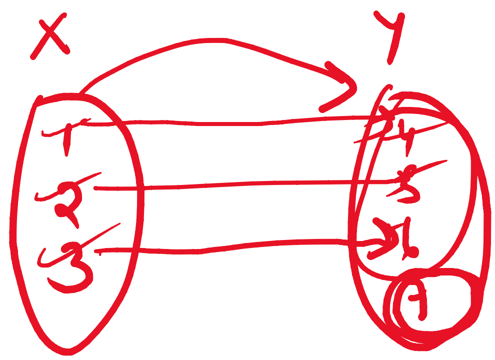
① One to One (Injective)



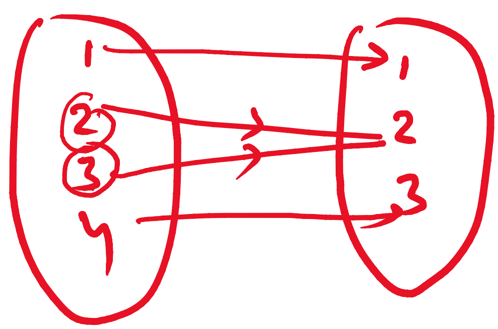
② Onto mapping (surjective)

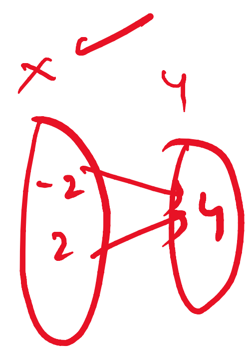


③ Into mapping (Bijective)



④ Many-one





Properties of functions:

1. $X = \phi$

$\Rightarrow f(X) = \phi$

$\Rightarrow f(x) = \phi$

Null set has no image.

2. Let $A, B, C \subset X$ | $A \subseteq B$, then $f(A) \subseteq f(B)$

f: square of a

Let $A = \{1, 2, 3\}$ $\quad B = \{1, 2, 3, 4\}$

$f(A) = \{1, 4, 9\}$ $\quad f(B) = \{1, 4, 9, 16\}$

$A \subset B \Rightarrow f(A) \subset f(B)$

Proved

3. $\forall A, B \subseteq X$

then $f(A) \cup f(B) = f(A \cup B)$

$A = \{1, 2, 3\}$ $\quad B = \{1, 2, 3, 4\}$

$f(A) = \{1, 4, 9\}$ $\quad f(B) = \{1, 4, 9, 16\}$

$f(A) \cup f(B) = \{1, 4, 9, 16\}$ — (1)

$A \cup B = \{1, 2, 3, 4\}$

$f(A \cup B) = \{1, 4, 9, 16\}$ — (2)

$\therefore f(A \cup B) = f(A) \cup f(B)$

4. $\forall A, B \subseteq X$, $f(A \cap B) \subseteq f(A) \cap f(B)$.

$A = \{1, 2, 3\}$ $\quad$ $B = \{1, 2\}$

$A \cap B = \{1, 2\}$

$f(A) = \{1, 4, 9\}$

$f(B) = \{1, 4\}$

$f(A) \cap f(B) = \{1, 4\}$

$f(A \cap B) = \{1, 4\}$

$f(A \cap B) = f(A) \cap f(B)$

$\subseteq$

Proof of Property 2

$\boxed{A, B \subseteq X \text{ then } f(A) \subseteq f(B)}$

x $\quad$ A $\{1 \quad 2 \quad 3\}$

x' $f(A)$ $\{1, 4, 9\}$

Let $A, B \neq \phi \subseteq X$

$f : X \to Y$

$f(A) = \{f(x) : x \in A\}$

$f(B) = \{f(x) : x \in B\}$

Given $A \subseteq B$ we have to show $f(A) \subseteq f(B)$

$f(x') \in f(A)$ — (1)

such $x' \in A$

Since $A \subseteq B$

Since $A \subseteq B$

$x' \in B$

(operating on both sides) $\Rightarrow f(x') \in f(B)$ — (2)

From (1) and (2) $f(A) \subseteq f(B)$

Proved

$x \in f(A) \cup f(B)$

$x \in f(A)$ or $x \in f(B)$

there exist $y \in A \cup B$

so that $x = f(y)$

or $x \in f(A \cup B)$

Now

if $x \in f(A \cup B)$

there exist $y \in A \cup B$

such that $f(x) = y$