Q. $f(x) = \begin{cases} \big|\, |x-1| - 1 \,\big| , & x < 1 \\ [x] , & x \geq 1 \end{cases}$

Then the points of discontinuity of $f(x)$ is:

(a) all integers ≥ 0

(b) all integers ≥ 1

(c) All integer > 1

(d) only $x = 1$.

$f(x) = \big|\, |x-1| - 1 \,\big| , \; x < 1$

$= \big| -(x-1) - 1 \big|$

$= \big| -x + \cancel{1} - \cancel{1} \big| = |-x| = |x|$.

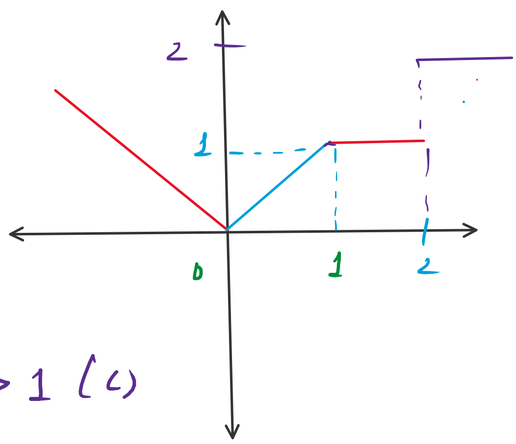
$|x-1| = (x-1), \; x \geq 1$

$\quad\quad -(x-1), \; x < 1$

$f(x) = \begin{cases} |x|, & x < 1 \\ [x], & x \geq 1 \end{cases}$

$|x| = \begin{cases} x, & x \geq 0. \\ -x, & x < 0. \end{cases}$

$f(x) = \begin{cases} -x, & x < 0. \\ x, & 0 \leq x < 1. \\ [x], & x \geq 1. \end{cases}$



At $x = 1$, $f(x) = [x] = 1 = x$.

$\therefore$ Discontinuous at all integeres > 1 (c)

Q. Let $f, g : [0, \infty) \to [0, \infty)$ be increasing and decreasing respectively. Define $h(x) = f[g(x)]$. If $h(0) = 0$ then $h(x) - h(1) = ?$

(a) Non-positive for $x \geq 1$, positive otherwise

(b) Always negative

(c) Always positive.

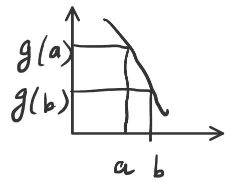
(d) Positive for $x \geq 1$, negative otherwise.

$f' > 0,\ g' < 0$.

$h(x) = f[g(x)]$.

$h' = \underbrace{f'}_{>0}\ \underbrace{g'}_{<0} < 0 \Rightarrow h(x)$ is a decreasing fn.

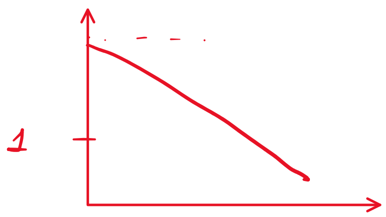
g: Decreasing fn



$a < b \Rightarrow g(a) > g(b)$

$h(x) - h(1)$ $< 0 \Rightarrow x > 1.$

$> 0 \Rightarrow x < 1$.



$x < 1 \Rightarrow h(x) > h(1)$

$x > 1 \Rightarrow h(1) > h(x)$

$\Rightarrow h(x) - h(1) > 0$ if $x < 1$.

$h(x) - h(1) < 0$ if $x > 1$.

Q. $\lim\limits_{t \to \infty} \dfrac{x^t - 1}{x^t + 1}$ $(x > 0)$ is:

(a) Limit exists.

(b) Limit exists & $= 1$.

(c) Limit exists & $= -1$.

(d) Limit does not exist.

$\boxed{0 < x < 1}$, $x^t \to 0$ as $t \to \infty$.

$\lim\limits_{t \to \infty} \dfrac{x^t - 1}{x^t + 1} = \dfrac{0 - 1}{0 + 1} = -1$.

$\boxed{x > 1}$, $x^t \to \infty$ as $t \to \infty$.

$\therefore \lim\limits_{t \to \infty} \dfrac{x^t - 1}{x^t + 1}$ $\left[\dfrac{\infty}{\infty}\right]$

Check whether limit exists for a fn $f(x)$ at $x = a$.

$\therefore$ Evaluate: $\lim\limits_{x \to a} f(x)$.

LHL: $\lim\limits_{x \to a^-} f(x) = l_1$

$\therefore \lim_{t\to\infty} \frac{x^t - 1}{x^t + 1} \quad \left[\frac{\infty}{\infty}\right]$

$\lim_{t\to\infty} \frac{1 - x^{-t}}{1 + x^{-t}} = \frac{1-0}{1+0} = 1$

LHL : $\lim_{x\to a^-} f(x) = l_1$

RHL : $\lim_{x\to a^+} f(x) = l_2$

$l_1 \neq l_2$

$\therefore$ Limit does not exist

Q. The maximum value of $f(a) = \int_{a-1}^{a+1} e^{-(x-1)^2} dx$, $a \in \mathbb{R}$ is attained at $a =$

(a) 0 (b) 1 (c) −1 (d) 2 .

$$f(a) = \int_{a-1}^{a+1} e^{-(x-1)^2} dx$$

$f'(a) = 0 \Rightarrow$ To find $f'(a)$ use Leibnitz Theorem.

$$f'(a) = e^{-(a+1-1)^2} \cdot 1 - e^{-(a-1-1)^2} \cdot 1 .$$

$$= e^{-a^2} - e^{-(a-2)^2} .$$

$$= e^{-a^2} - e^{-(a^2-4a+4)}$$

$$= e^{-a^2} - e^{-a^2+4a-4} .$$

$$= e^{-a^2}\left[1 - e^{4a-4}\right] .$$

$f'(a) = 0 \Rightarrow e^{-a^2}\left[1 - e^{4(a-1)}\right] = 0$

$e^{-a^2} \neq 0 \Rightarrow 1 - e^{4(a-1)} = 0$

$\Rightarrow 1 = e^{4(a-1)}$

$\Rightarrow e^0 = e^{4(a-1)}$

$\Rightarrow 0 = 4(a-1) \Rightarrow a = 1$

Q. Let f be a fn s.t $f(x) \cdot f'(x) < 0 \; \forall x \in \mathbb{R}$. Then :

Q. Let f be a fn s.t $f(x) \cdot f'(x) < 0 \ \forall x \in \mathbb{R}$. Then:

(a) $f(x)$ is increasing fn

(b) $f(x)$ is decreasing fn

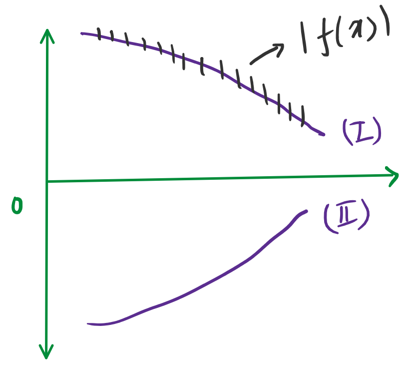
(c) $|f(x)|$ is increasing fn

(d) $|f(x)|$ is decreasing fn.

$f(x) \cdot f'(x) < 0$.

$\Rightarrow$ If $f(x) > 0 \Rightarrow f'(x) < 0$

or if $f(x) < 0 \Rightarrow f'(x) > 0$

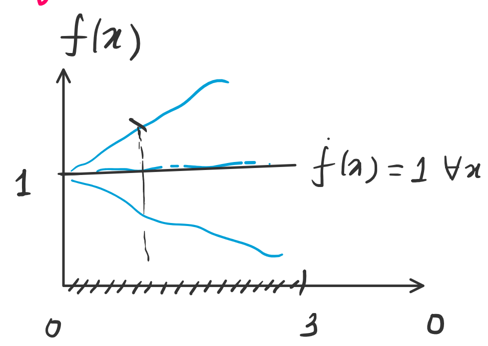


Q. Let $f(x)$ be continuous over $[0,3]$ with $f(0) = 1$. If $f(x)$ takes only rational values in $[0,3]$, then $f(1) =$

(a) 1 (b) Any +ve real Number

(c) 0 (d) cannot say.

$f(x) = 1 \ \forall x \in [0,3]$.



HW.

Q. The no. of disjoint intervals over which $f(x) = \left|0.5x^2 - |x|\right|$ is decreasing is: (a) 1 (b) 2

(c) 3 (d) None.