15 February 2024 14:39

$a_1 \sin^n x + a_2 \sin^{n-1} x \cos x + \dots$

Solve $2\sin^2 x - 5\sin x \cos x - 8\cos^2 x = -2$.

Divide by $\cos^2 x$.

$2\tan^2 x - 5\tan x - 8 = -2\sec^2 x$

$2\tan^2 x - 5\tan x - 8 = -2(1+\tan^2 x)$

$4\tan^2 x - 5\tan x - 6 = 0$

$4\tan^2 x - 8\tan x + 3\tan x - 6 = 0$

$4\tan x(\tan x - 2) + 3(\tan x - 2) = 0$

$(\tan x - 2)(4\tan x + 3) = 0$

$\tan x = 2, -3/4$

$x = n\pi + \arctan(2)$,

$n\pi + \arctan(-3/4)$

Solve the equation $(1-\tan\theta)(1+\sin 2\theta) = 1+\tan\theta$.

$\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

$\sin 2\theta = \dfrac{2\sin\theta\cos\theta}{1}$

$= \dfrac{2\tan\theta}{\sec^2\theta}$

$$(1-\tan\theta)\left(1+\frac{2\tan\theta}{1+\tan^2\theta}\right) = 1+\tan\theta$$

$$(1-\tan\theta)\frac{(1+\tan\theta)^2}{1+\tan^2\theta} = (1+\tan\theta)$$

$$\frac{(1-\tan\theta)(1+\tan\theta)^2}{1+\tan^2\theta} - (1+\tan\theta) = 0.$$

$$(1+\tan\theta)\left[\frac{(1-\tan\theta)(1+\tan\theta)}{1+\tan^2\theta} - 1\right] = 0.$$

$$(1+\tan\theta)\left[\frac{1-\tan^2\theta - (1+\tan^2\theta)}{1+\tan^2\theta}\right] = 0.$$

$$\frac{(1+\tan\theta)(-2\tan^2\theta)}{1+\tan^2\theta} = 0.$$

$\tan\theta = 0, -1$

$\theta = n\pi, \; n\pi + \arctan(-1)$

$= n\pi, \; n\pi + \frac{3\pi}{4}.$

$\sin 2\theta, \cos 2\theta, \tan 2\theta \to \tan\theta.$

$\sin\theta, \cos\theta \to \cos 2\theta.$

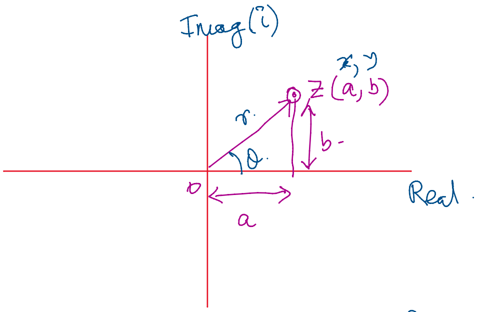
COMPLEX NUMBERS

$z = a + bi$ $\quad i = \sqrt{-1}, \; (a, b) \in \mathbb{R}.$

Imag(i)

$r = |z|$

$Z = a + bi$ $\quad$ $i = \sqrt{-1}$, (a, b)



$r = |z|$

$r^2 = a^2 + b^2$

$|z| = \sqrt{a^2+b^2}$

locus of $|z|$

$|z| = \sqrt{x^2+y^2}$

$|z|^2 = x^2+y^2$

$x^2+y^2 = r^2 \rightarrow$ circle with center as origin and radius $= |z|$

$x = r\cos\theta$

$\tan\theta = \frac{y}{x}$ $\qquad$ $y = r\sin\theta$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$Z = x + yi$

$Z = r\cos\theta + r\sin\theta\, i$

$Z = r(\cos\theta + i\sin\theta)$ $\qquad$ $\underline{cis\theta} = \cos\theta + i\sin\theta$

<u>Taylor Series</u> $\rightarrow$ infinite series which is used to represent $\sin x, \cos x, e^x, \log_e(1+x)$ etc as polynomials. (power series)

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{i\theta} = 1 + \frac{i\theta}{1!} + \frac{i^2\theta^2}{2!} + \frac{i^3\theta^3}{3!} + \frac{i^4\theta^4}{4!} + \frac{i^5\theta^5}{5!} + \frac{i^6\theta^6}{6!} + \cdots$$

$i = \sqrt{-1}$ $\quad$ $i^2 = -1$ $\quad$ $i^3 = i^2 \cdot i = -i$ $\quad$ $i^4 = 1$ $\quad$ $i^5 = i$

$i, -1, -i, 1$

$$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2!} - \frac{i\theta^3}{3!} + \frac{\theta^4}{4!} + \frac{i\theta^5}{5!} - \frac{\theta^6}{6!} + \cdots$$

$$= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots\right) + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right)$$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

$Z = r(\cos\theta + i\sin\theta) = r.e^{i\theta}$

$z^2 = (re^{i\theta})^2 = r^2 e^{i2\theta} = r^2(\cos 2\theta + i\sin 2\theta)$

$z^n = r^n(\cos n\theta + i\sin n\theta) = r^n cis(n\theta) = |z|^n cis(n\theta)$

$$\boxed{(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta}$$

$e^{i\theta} = cis\,\theta \rightarrow$ multiply or divide complex numbers

$\boxed{z_1 = 2+3i}$ $\quad z_2 = -4+2i \quad z_3 = 3-4i$ $\quad \dfrac{z_1 z_2}{z_3} = ?$

$z_1 = r_1 e^{i\theta_1} \quad z_2 = r_2 e^{i\theta_2} \quad z_3 = r_3 e^{i\theta_3}$.

$0.4\sqrt{65}$

$$\frac{z_1 z_2}{z_3} = \left(\frac{r_1 r_2}{r_3}\right) e^{i(\theta_1+\theta_2-\theta_3)} = \frac{2\sqrt{65}}{5} e^{i(-98^\circ)} = 3.2 e^{i(-98^\circ)}$$

$$= 3.2\left[\cos(-98^\circ) + i\sin(-98^\circ)\right]$$

$$= 3.2\left[\cos 98^\circ - i\sin 98^\circ\right]$$

$r_1 = \sqrt{2^2+3^2} = \sqrt{13}$ $\quad r_2 = \sqrt{16+4} = 2\sqrt{5}$ $\quad r_3 = \sqrt{9+16} = 5$

$\theta_1 = \tan^{-1}\left(\frac{3}{2}\right)$ $\quad \theta_2 = \tan^{-1}\left(\frac{1}{-2}\right)$ $\quad \theta_3 = \tan^{-1}\left(\frac{-4}{3}\right) = -53^\circ$

$\cos(90^\circ+\theta) = -\sin\theta$.

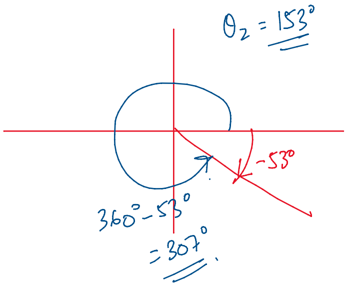
$\sin(90+\theta) = \cos\theta$

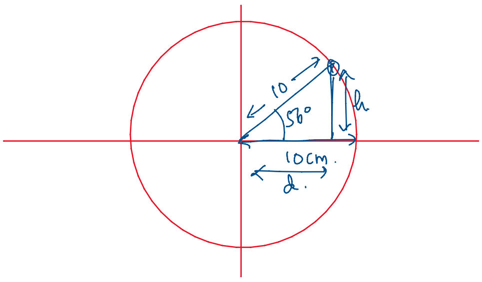
$\theta_1 = 56^\circ$ $\quad \theta_2 = 153^\circ$ $\quad \theta_3 = 360 - 53^\circ = 307^\circ$.

$= 3.2\left[-\sin 8^\circ - i\cos 8^\circ\right]$

$-98^\circ = -98 \times \frac{\pi}{180}$ rad.

$1^\circ = \frac{\pi}{180}$ rad.





$\tan 56^\circ = \frac{h}{d}$.

$$\sqrt{65} = 8 + \frac{1}{2} \times \frac{(65-64)}{8} = 8 + \frac{1}{16} = 8.0$$

$\sqrt{64}$