Questions

72. The number of integer (positive, negative or zero) solutions of

$$xy - 6(x + y) = 0$$

with $x \leq y$ is

(A) 5; (B) 10; (C) 12; (D) 9.

$xy - 6x - 6y = 0 \Rightarrow xy - 6x - 6y + 36 = 36$

$\Rightarrow (x-6)(y-6) = 36$

Factors of 36 $\rightarrow$ 1, 2, 3, 4, 6, 9, 12, 18, 36

$x-6=1,\ y-6=36$

$x-6=2,\ y-6=18$

$\vdots$

$(x,y) = (7,42), (8,24), (9,18), (10,15), (12,12)$

76. Let n be a *positive* integer. Now consider all numbers of the form $3^{2n+1} + 2^{2n+1}$. Only one of the following statements is true regarding the *last digit* of these numbers. Which one is it?

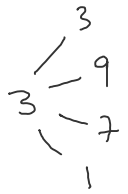
(A) It is 5 for some of these numbers but not for all.
(B) It is 5 for all these numbers.
(C) It is always 5 for $n \leq 10$ and it is 5 for some $n > 10$.
(D) It is odd for all of these numbers but not necessarily 5.

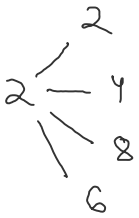
$n=0,\ 3^1 + 2^1 = 5$

$n=1,\ 3^3 + 2^3 = \ldots 7 + \ldots 8$

$n=2,\ 3^5 + 2^5 = \ldots 3 + \ldots 2$

$n=3,\ 3^7 + 2^7 = \ldots 7 + \ldots 8$





Functions

$|x| + |x-3| > 3$

$x < 0$: $-x-(x-3) > 3$

$0 \le x < 3$: $x-(x-3) > 3$

$x \ge 3$: $x+(x-3) > 3$

$\frac{1}{|x|-3} < \frac{1}{2}$

$\frac{1}{-x-3} < \frac{1}{2}$

$\frac{1}{x-3} < \frac{1}{2}$

$\log_{3x+5}(9x^2+8x+8) > 2$

$3x+5 > 0$ → $3x+5 > 1$

→ $0 < 3x+5 < 1$

$3x+5 > 1$

$9x^2+8x+8 > (3x+5)^2 \Rightarrow x < -\frac{17}{22}$

$3x+5 > 1 \Rightarrow x > -4/3$

$-4/3 < x < -17/22$ ✓

$0 < (3x+5) < 1$

$9x^2+8x+8 < (3x+5)^2 \Rightarrow x > -\frac{17}{22}$

$3x+5 > 0 \Rightarrow x > -5/3$

$3x+5 < 1 \Rightarrow x < -4/3$

$x > -17/22$ & $x \in (-5/3, -4/3)$ no sol

$\therefore x \in (-4/3, -17/22)$

Algebra of func

$f: D_1 \to R$

$g: D_2 \to R$

$(f+g)x = f(x) + g(x)$

$f+g : D_1 \cap D_2 \to R$

$f-g$:

$f \cdot g$:

$f/g : D_1 \cap D_2 \to \{R - g(x) = 0\}$

$f(x) = \sqrt{6-x}$

$g(x) = \sqrt{x-2}$

$(f+g)(x) = \sqrt{6-x} + \sqrt{x-2}$

$[2, 6]$

$f(x) = \frac{1}{\sqrt{x-[x]}}$. domain?

$f(x) = \cos^{-1}(\log_3 x)$

$x \in [1/3, 3]$

Functions

$f(x) = \begin{matrix} 3x-1 \\ 20-4x \end{matrix}$ $\quad 3x-1 \geq 20-4x$

$\Rightarrow x \geq 3$

$3x-1 \geq 0 \quad \& \quad 20-4x \geq 0$

$x \geq 1/3 \qquad x \leq 5$

$x \in [1,\infty), \quad x \in [1,5], \quad x \in [3,\infty)$

$\Rightarrow \qquad x \in [3,5] \qquad x \in \{3,4,5\}$

HW $\quad f(x) = \log_x 5 + \sqrt{\cos(\sin x)}$