30 January 2024 21:11

Cauchy's Mean Value Theorem :-

Consider 2 fns $f(x)$ and $g(x)$ s.t:

(i) Both are continuous in $[a,b]$.

(ii) Both are differentiable in (a,b).

(iii) $g'(x) \neq 0 \ \forall x \in (a,b)$

Then $\exists$ a pt 'c' $\in (a,b)$ s.t $\dfrac{f(b)-f(a)}{g(b)-g(a)} = \dfrac{f'(c)}{g'(c)}$

LMVT

$f(x)$ cont in $[a,b]$ ✓

$f(x)$ diff in (a,b) ✓

$\exists$ a pt 'c' $\in (a,b)$ s.t

$\dfrac{f(b)-f(a)}{(b-a)} = f'(c)$

$\dfrac{g(b)-g(a)}{(b-a)} = g'(c)$

Q. Let $f(x)$ and $g(x)$ be differentiable fns in $[0,1]$ s.t $f(0)=2$, $g(0)=0$, $f(1)=6$. Let $\exists$ a no. 'c' $\in (0,1)$ s.t $f'(c) = 2g'(c)$. Then $g(1) = ?$

(a) 1 (b) 2 (c) -2 (d) -1

Q. Let $f(x) = \min\{x+1, |x|+1\}$. Then:

(a) $f(x)$ is diff everywhere

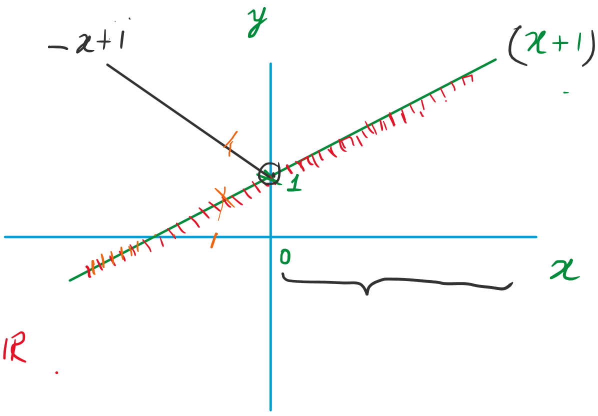
(b) $f(x)$ is not diff at $x=0$

(c) $f(x) \geq 1 \ \forall x \in \mathbb{R}$

(d) $f(x)$ is not diff at $x=1$

$f(x) = \min\{x+1, |x|+1\}$

$|x|+1 = \begin{cases} x+1, & x \geq 0 \\ -x+1, & x < 0 \end{cases}$



$\min\{x+1, |x|+1\} = (x+1) \ \forall x \in \mathbb{R}$

Q. Let $f : \underbrace{[-7,0]}_{x} \to \mathbb{R}$ be continuous on $[-7,0]$ and diff on $(-7,0)$. If $f(-7) = -3$ and ...

... be continuous on $[-7,0]$ and diff on $(-7,0)$. If $f(-7) = -3$ and $f'(x) \leqslant 2 \ \forall \ x \in (-7,0)$, then the interval in which $f(-1) + f(0)$ lies is :-

~~(a)~~ $(-\infty, 20]$ (b) $[-3, 11]$ (c) $(-\infty, 11]$ (d) $[-6, 20]$

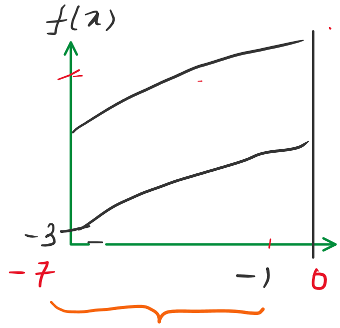
LMVT: $[-7, 0]$.

$$\frac{f(0) - f(-7)}{0-(-7)} = f'(c) \text{ for } c \in (-7,0).$$
$$\hookrightarrow \leqslant 2.$$

$f'(x) \leq 2$

$\int f'(x)\,dx \leq \int 2\,dx$

$f(x) \leq (2x + c)$



$$\Rightarrow \frac{f(0) - f(-7)}{7} \leqslant 2.$$

$$\Rightarrow \frac{f(0) + 3}{7} \leqslant 2 \Rightarrow f(0) \leqslant 11.$$

LMVT: $[-7, -1]$.

$$\Rightarrow \frac{f(-1) - f(-7)}{-1+7} \leqslant 2.$$

$\Rightarrow f(-1) \leqslant 9$. $\Rightarrow f(0) + f(-1) \leqslant 20$.

Q. Let $a, b \in \mathbb{R}$ and $f(x) = \ln|x| + bx^2 + ax, x \neq 0$ have extreme values at $x = -1$ and $x = 2$. Then:

(a) $f(x)$ has local minima at $x=-1$, local max at $x = 2$

(b) $f(x)$ has local max at $x = -1$, local min at $x = 2$

(c) $f(x)$ has local min at $x=-1$ & $x = 2$

~~(d)~~ $f(x)$ has local max at $x = -1$ & $x = 2$.

$$f(x) = \ln|x| + bx^2 + ax$$

$f'(x) = \frac{1}{x} + 2bx + a. \Rightarrow f''(x) = -\frac{1}{x^2} + 2b = -\frac{1}{x^2} - \frac{1}{4}.$

$f'(-1) = 0$ and $f'(2) = 0$.

$-1 - 2b + a = 0 \qquad \frac{1}{2} + 4b + a = 0. \Rightarrow a = 1/2, \; b = -1/4.$

$$\therefore f'(x) = \frac{1}{x} - \frac{x}{2} + \frac{1}{2} = \frac{2 - x^2 + x}{2x} = -\frac{(x^2 - x - 2)}{2x}$$

$$= -\frac{(x+1)(x-2)}{2x}.$$

HW.

Q. Let $f: \mathbb{R} \to \mathbb{R}$ be defined as: $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$. Then:

(a) $f(x)$ is continuous $\forall x$

(b) discontinuous only at $x = 0$

(c) discontinuous at all integers

(d) continuous only at $x = 0$.