04 December 2023 08:05 PM

$n_1 = 55$ $n_2 = 45$ $N = 100$

$\bar{x}_1 = 6.6$ $\bar{x}_2 = ?$ $\bar{x} = 6.5$

$N\bar{x} = n_1\bar{x}_1 + n_2\bar{x}_2$

$100 \times 6.5 = 55 \times 6.6 + 45\bar{x}_2$

$65 = 55 \times 6.6 + 45\bar{x}_2$

$\bar{x}_2 = \dfrac{65 - 55 \times 6.6}{45}$

$= \dfrac{287}{45} = 6.38$

$\sigma_2^2 = 3.89$

$\sigma_2 = \sqrt{3.89}$
$= 1.97$

— * —

NG Das : Solve all examples

Correlation coefficient between X and Y.

$Y = f(X)$

dep indep

$$r_{xy} = \frac{\text{cov}(x, y)}{\sqrt{V(x)}\sqrt{V(y)}}$$

① $-1 \le r \le 1$

② $cov(x,y) = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})$

or, $\boxed{cov(x,y) = \frac{1}{n}\sum XY - \bar{X}\bar{Y}}$

Q $n = 25$ $\quad \Sigma x = 125$ $\quad \Sigma y = 100$

$\Sigma x^2 = 650$ $\quad \Sigma y^2 = 460$ $\quad \Sigma xy = 508$

(x, y) were copied $(6, 14)$ and $(8, 6)$ while correct $(8, 12)$ and $(6, 8)$ resp.

$n = 25$ $\quad \Sigma x = 125$ $\quad \bar{x} = \Sigma x / n$

correct $\Sigma x = 125 - \cancel{6} - \cancel{8} + \cancel{8} + \cancel{6}$

$\Sigma x' = 125$, $n = 25$

new mean $\bar{x}' = \frac{125}{25} = 5$

new mean $\bar{x}$ = $\frac{\quad}{25}$

$\sum x^2 = 650$

new $\sum x^2 = 650 - \cancel{6^2} - \cancel{8^2} + \cancel{8^2} + \cancel{6^2}$

$= 650$

$\therefore$ new variance $\sigma^2 = \frac{1}{n}\sum x^2 - \bar{x}^2$

$= \left(\frac{1}{25} \times 650\right) - 5^2$

$= 26 - 25$

$= 1$

$s.d(x) = \sqrt{1} = 1$.

$\sum y = 100$ new $\sum y = 100 - (14) - (6) + 12 + 8$

$= 100$

new mean $\bar{y}' = \frac{\sum y}{n} = \frac{100}{25} = 4$

$\sum y^2 = 460$

new $\sum y^2 = 460 - (14)^2 - (6)^2 + (12)^2 + (8)^2$

$= 436$

$$\sigma_y = \sqrt{\frac{1}{n}\Sigma y^2 - \bar{y}^2}$$

$$= \sqrt{\frac{436}{25} - 4^2}$$

$$= \sqrt{1.44}$$

$$= 1.2$$

$$r_{xy} = \frac{cov}{\sigma_x \sigma_y}$$

$$cov(x,y) = \frac{1}{n}\Sigma xy - \bar{x}\bar{y}$$

given $\Sigma xy = 508$

$$\text{new } \Sigma xy = 508 - (6 \times 14) - \cancel{(8 \times 6)} + (8 \times 12) + \cancel{(6 \times 8)}$$

$$= 508 - 84 + 96$$

$$= 508 + 12$$

$$= 520$$

$$\therefore cov(x,y) = \frac{1}{n}\Sigma xy - \bar{x}\bar{y}$$

$$= \frac{520}{25} - 5 \times 4$$

$$= \frac{520}{25} - 20$$

$$= \frac{520 - 500}{25} = \frac{20}{25} = 4/5$$

$$= 0.8$$

$$r_{x,y} = \frac{cov(x,y)}{\sigma_x \sigma_y} = \frac{0.8}{1 \times 1.2}$$

$$= \frac{0.8}{1.2} = \frac{8}{12} = \frac{2}{3}$$

(ans)

Q Find the correlation coefficient of x and y using the following data:

x	y	x^2	y^2	xy
1	6	1	36	6
2	7	4	49	14
3	8	9	64	24
4	9	16	81	36
5	10	25	100	[illegible]

			81	36
5	10	25	100	50
$\Sigma x = 15$	$\Sigma y = 40$	$\Sigma x^2 = 55$	$\Sigma y^2 = 330$	$\Sigma xy = 120$

$$r = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

$$cov(x,y) = \frac{1}{n}\Sigma xy - \bar{x}\bar{y}$$

$\Sigma x \quad \Sigma y \quad \Sigma x^2 \quad \Sigma y^2 \quad \Sigma xy$

$$b_{yx} = \frac{cov(x,y)}{\sigma_x^2} = \frac{14}{2} = 7$$

$y - 8 = 7x - 21$

$\boxed{y - 7x = -13}$

$$\bar{x} = \frac{1}{n}\Sigma x = 15/5 = 3$$

$$\bar{y} = \frac{1}{n}\Sigma y = 40/5 = 8$$

$$\sigma_x = \sqrt{\frac{1}{n}\Sigma x^2 - \bar{x}^2} = \sqrt{\frac{55}{5} - 3^2}$$

$$= \sqrt{11 - 9}$$

$$= \sqrt{2}$$

$$= 1.41$$

Y on x: $y - \bar{y} = b_{yx}(x - \bar{x})$

$y - 8 = 7(x - 3)$

$$\sigma_y = \sqrt{\frac{1}{n}\Sigma y^2 - \bar{y}^2}$$

$$= \sqrt{\frac{1}{5} \times 330 - 8^2}$$

$$= \sqrt{\frac{330}{5} - 64}$$

$$= \sqrt{66 - 64}$$

$$= \sqrt{2}$$

$$cov(x,y) = \frac{1}{n}\Sigma xy - \bar{x}\bar{y}$$

$$= \frac{130}{5} - 5 \times 8$$

$$= \frac{130}{5} - 40$$

$$= -14$$

$$\therefore r_{x,y} = \frac{cov(x,y)}{\sqrt{V(x)}\sqrt{V(y)}} = \frac{-14}{\sqrt{2}\sqrt{2}}$$

$$= -14/2 = -7$$

$$-1 \le r \le 1$$

Q) Marks of 10 students in Mathematics and Statistics are given below,

Maths (x)	32	38	48	45	40	22	41	69	35	64
Statistics (y)	30	31	38	43	33	11	27	76	40	59

(i) Calculate the correlation coefficient between x and y.

(ii) also write the regression eqn of (y on x) and (x on y)

Regression Lines

1. Regression line equation of (y on x)
2. " equation of (x on y)

y on x: $(y-\bar{y}) = b_{yx}(x-\bar{x})$

or $\boxed{y = \bar{y} + b_{yx}(x-\bar{x})}$

indp- mean-$\bar{x}$

coefficients

indp-mean-x

dep

mean

regression coefficiente of y on x

Where regression coefficient $b_{y x} = \frac{cov(x,y)}{\sigma_x^2}$

Again X on Y :

$$x - \bar{x} = b_{xy}\,(y - \bar{y})$$

$$x = \bar{x} + b_{xy}\,(y - \bar{y})$$

Regression equation of x on y.

where $b_{xy} = \frac{cov(x,y)}{\sigma_y^2}$

and $r_{xy} = \sqrt{b_{xy} \times b_{yx}}$

or $r_{xy}^2 = b_{xy} \times b_{yx}$

ex:
reg coeff of y on x is 0.2
and reg coeff or x on y is 0.8
what is correlation coeff.

$b_{yx} = 0.2$
$b_{xy} = 0.8$
$r_{xy} = ?$

solution.

$$r_{xy} = \sqrt{b_{xy} \times b_{yx}}$$
$$= \sqrt{0.2 \times 0.8}$$
$$= \sqrt{0.16}$$
$$= 0.4 \text{ (ans)}$$