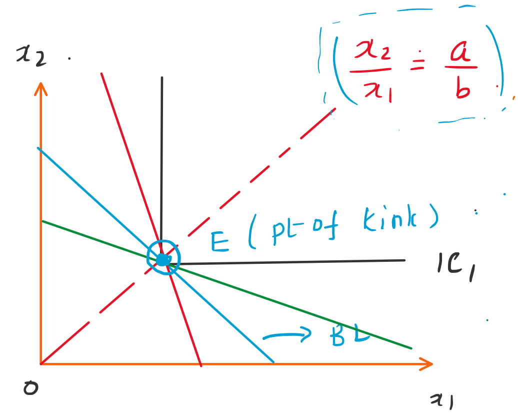
Perfect complements.

Q. $u = \min\{a x_1, b x_2\}$, $a, b > 0$, Find the marshallian demand curves for Good 1 & Good 2.

Good 1 & Good 2 are consumed in a fixed ratio

At optimal: $a x_1 = b x_2$

$\Rightarrow \boxed{\frac{x_2}{x_1} = \frac{a}{b}} \Rightarrow \boxed{x_2 = \frac{a}{b} x_1}$



$\therefore$ Budget line: $M = P_1 x_1 + P_2 x_2$

$M = P_1 x_1 + P_2 \frac{a}{b} \cdot x_1$

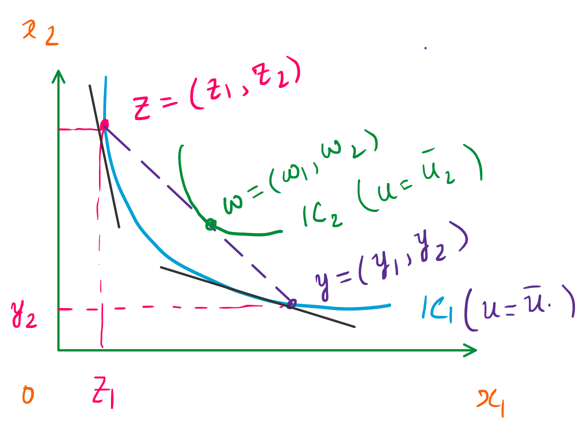
$M = x_1 \left(P_1 + \frac{a}{b} P_2 \right)$

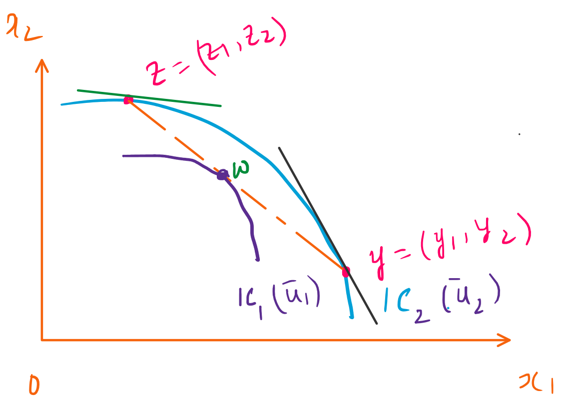
$M = x_1 \left(\frac{b P_1 + a P_2}{b} \right) \Rightarrow x_1^* = \frac{b \cdot M}{b P_1 + a P_2}$

$x_2^* = \frac{a}{b} \cdot x_1^* = \frac{a}{\cancel{b}} \cdot \frac{\cancel{b} M}{b P_1 + a P_2} = \frac{a M}{b P_1 + a P_2}$

$\Rightarrow$ Marshallian demands.

Convex Preferences and Concave Preferences:





w = Average bundles.

y, z = Extreme bundles.

Convex pref ⇒ Avg is preferred to extreme. (Interior soln)

Concave pref ⇒ Extremes are preferred to average.

} Economic interpretation.

Concave pref $\Rightarrow$ Extremes are preferred to average. | interpretation.

↳ (Corner soln).

In general: $u = u(x_1, x_2)$.

Diff: $du = \left(\frac{\partial u}{\partial x_1}\right) dx_1 + \left(\frac{\partial u}{\partial x_2}\right) dx_2$

For IC, $du = 0 \Rightarrow 0 = MU_1 \cdot dx_1 + MU_2 \cdot dx_2$.

$$\left.\frac{dx_2}{dx_1}\right|_{IC} = -\frac{MU_1}{MU_2}.$$

$$MRS = \left| \left.\frac{dx_2}{dx_1}\right|_{IC} \right| = \frac{MU_1}{MU_2}.$$

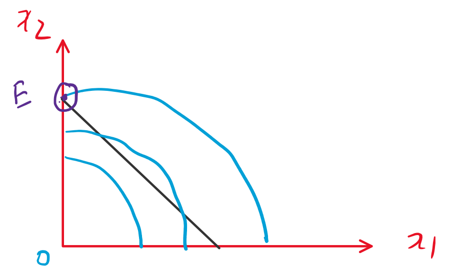
For convex IC's: MRS ↓ as x_1 ↑. $\Rightarrow \frac{\partial MRS}{\partial x_1} < 0$ For convex IC's

For concave IC's: MRS ↑ as x_1 ↑ $\Rightarrow \frac{\partial MRS}{\partial x_1} > 0$ For concave IC's.

Q. $u = x_1^2 + x_2^2$. Find the marshallian demand curves.

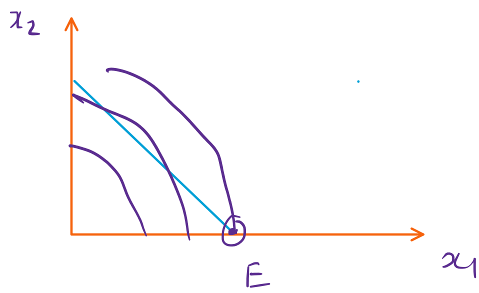
$$MRS = \frac{MU_1}{MU_2} = \frac{\partial u/\partial x_1}{\partial u/\partial x_2} = \frac{2x_1}{2x_2} = \frac{x_1}{x_2}.$$

$$\frac{\partial MRS}{\partial x_1} = \frac{1}{x_2} \geq 0 \Rightarrow \text{concave pref.}$$

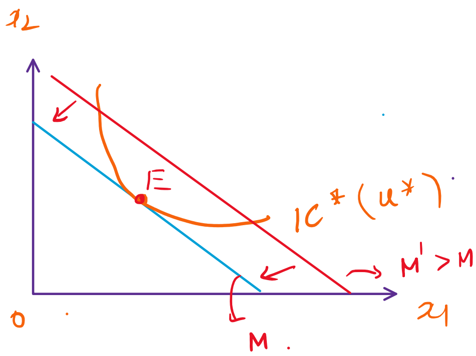


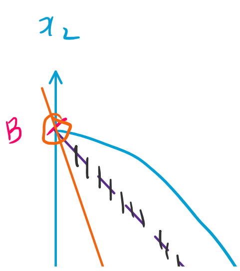
Two ways of vizualizing the optimal point graphically:

(i) Fix the BL & keep the highest possible IC on the right to reach E.



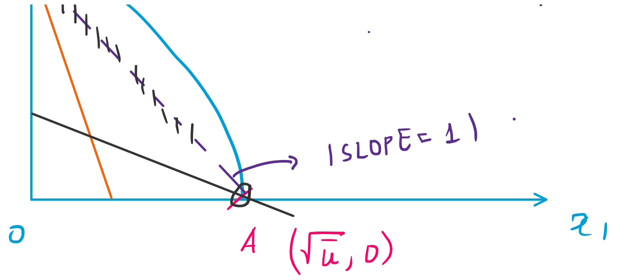
(ii) Fix IC^*, & take the leftmost B.L to reach pt E.





Fix $\bar{u} = x_1^2 + x_2^2$

M .



For A, $(\sqrt{\bar{u}}, 0)$

For B, $(0, \sqrt{\bar{u}})$

$|\text{Slope of BL}| = \frac{P_1}{P_2}$

i) If $\frac{P_1}{P_2} > 1 \Rightarrow$ opt pt B $\Rightarrow x_1^* = 0$

ii) If $\frac{P_1}{P_2} < 1 \Rightarrow$ opt pt A $\Rightarrow x_2^* = 0$

iii) If $\frac{P_1}{P_2} = 1 \Rightarrow$ opt is either pt A / pt B.

i) If $\left(\frac{P_1}{P_2} > 1\right) \Rightarrow P_1 > P_2$ then $x_1^* = 0$, $x_2^* = \frac{M}{P_2}$

ii) If $\left(\frac{P_1}{P_2} < 1\right) \Rightarrow P_1 < P_2$ then $x_2^* = 0$, $x_1^* = \frac{M}{P_1}$

$\rightarrow$ Buying the good that is cheaper

iii) If $\left(\frac{P_1}{P_2} = 1\right) \Rightarrow P_1 = P_2$ then either $\left(\frac{M}{P_1}, 0\right)$ or $\left(0, \frac{M}{P_2}\right)$.

HW.

Q. $u = \max\{x_1, x_2\}$. Plot the IC's.