1. Euler's Theorem.
2. Elasticity of substitution.
3. Elasticity of Demand.
4. Application in Economics.

Questions:

1. Check degree of homogeneity and verify Euler's theorem for Cobb-Douglas production function.

H.W. 2. $q = f(L, K)$

$q = 75\left[0.3K^{-0.4} + 0.7L^{-0.4}\right]^{-2.5}$ verify Euler's theorem.

(1) Find degree of homogeneity $= 1$

(2) $\partial q/\partial L \cdot L + \partial q/\partial K \cdot K = q$.

3. Evaluate the elasticity of substitution for the production fn $z = Ax^{\alpha}y^{1-\alpha}$; A, α const $0 < \alpha < 1$.

H.W. 4. Find elasticity of substitution for $q = \left[aK^{-b} + (1-a)L^{-b}\right]^{-\frac{1}{b}}$

$\frac{MP_L}{MP_K} \Rightarrow \log \Rightarrow d\log$ (ans $\sigma = \frac{1}{1-b}$).

5. Total Revenue from sale of x is $R = 100Q - 2Q^2$. Calculate point elasticity of demand $|e_p|$ when $MR = 20$.

6. A consumer's demand curve for x is $P = 100 - \sqrt{Q}$. Calculate point elasticity of demand when price of $Q = 60$.

$|e_p| = \frac{dQ}{dP} \times \frac{P}{Q}$

$\sqrt{Q} = 100 - P$

$\frac{-1}{2\sqrt{Q}} \cdot \frac{dQ}{dP} = -1$

$\frac{dQ}{dP} = 2\sqrt{Q}$

$|e_p| = \frac{2\sqrt{Q} \times (100 - \sqrt{Q})}{Q}$

$= \frac{200\sqrt{Q} - 2Q}{Q}$

$= \frac{200}{\sqrt{Q}} - 2$

H.W. 7. $X_1 = 300 - 0.5P_1^2 + 0.02P_2 + 0.05Y$. Find income elasticity at $P_1 = 12$, $P_2 = 10$ and $Y = 200$.

$e_m = \frac{dX_1}{dY} \cdot \frac{Y}{X_1}$

$= 0.05 \times \frac{200}{(\ )} =$ ans $= 0.04$.

8. The production fn $q = K^2 - 3KL + 4L^2$

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Find max amount of capital that can be employed when 7 units of Labour are employed.

$= \frac{200}{\sqrt{Q}} - 2$

$\frac{200}{\sqrt{60}} - 2$

(10) If the production fn $U = 8x_1^{1/2} + 20x_2^{1/2}$ and if $P_1 = 1$ and $P_2 = 5$. Derive the equation of ICC.

Euler's Theorem:

$$z = f(y, x)$$

Let fn z be a homogenous fn of degree 'n'.
then Euler's Theorem states that,

$$\frac{\partial f}{\partial y} \cdot y + \frac{\partial f}{\partial x} \cdot x = n \cdot Z = n f(x, y)$$

Soln 1: Cobb-Douglas Production fn is given by.

$$Q = L^{\alpha} \cdot K^{\beta}$$

Let us change 'L' and 'K' by λ proportion,

Then, $(\lambda L)^{\alpha} (\lambda K)^{\beta} = \lambda^{\alpha+\beta} (L^{\alpha} \cdot K^{\beta}) = \lambda^{\alpha+\beta} \cdot Q$

$\therefore$ The degree of homogeneity is $(\alpha+\beta)$.

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to prove Euler's theorem we have to show.

$$\boxed{\frac{\partial Q}{\partial L}\cdot L + \frac{\partial Q}{\partial K}\cdot K = (\alpha+\beta)Q}$$

Now, $Q = L^{\alpha} K^{\beta}$

LHS : $\frac{\partial Q}{\partial L}\cdot L + \frac{\partial Q}{\partial K}\cdot K$

$$= \alpha L^{\alpha-1}\cdot K^{\beta}\cdot L + L^{\alpha}\beta K^{\beta-1}\cdot K$$

$$= \alpha L^{\alpha}K^{\beta} + \beta L^{\alpha}K^{\beta}$$

$$= (\alpha+\beta)\left(\underline{L^{\alpha}K^{\beta}}\right)$$

$$= (\alpha+\beta)Q = \text{RHS}.$$

Verified

Elasticity of Substitution, $\sigma = \dfrac{\text{prop change in } (K/L)}{\text{prop chnge in } (MRTS_{L,K})}$

$$= \frac{\frac{d(K/L)}{K/L}}{\frac{d(MRTS_{L,K})}{MRTS_{L,K}}}$$

$$\sigma = \frac{d\log(K/L)}{d\log(MRTS_{L,K})}$$

$$\sigma = \frac{d\log(K/L)}{d\log(MP_L/MP_K)}$$

soln.
(2)

soln.

3

$$Z = A x^{\alpha} y^{1-\alpha} \quad ; \quad \sigma = \frac{d \log (y/x)}{d \log \left(\frac{Z_x}{Z_y}\right)}$$

$$Z_x = \partial Z / \partial x = A \alpha x^{\alpha-1} y^{1-\alpha}$$

$$Z_y = \partial Z / \partial y = A x^{\alpha} (1-\alpha) y^{-\alpha}$$

$$\frac{Z_x}{Z_y} = \frac{\cancel{A} \alpha x^{\alpha-1} y^{1-\alpha}}{\cancel{A} (1-\alpha) x^{\alpha} y^{-\alpha}} = \frac{\alpha}{(1-\alpha)} \cdot \frac{y}{x}$$

$$\log \left(\frac{Z_x}{Z_y}\right) = \log \left(\frac{\alpha}{1-\alpha}\right) + \log (y/x)$$

Differentiating on both sides.

$$d \log \left(\frac{Z_x}{Z_y}\right) = 0 + d \log (y/x)$$

$$\text{or,} \quad \frac{d \log (y/x)}{d \log (Z_x / Z_y)} = 1$$

$$\text{or,} \quad \boxed{\sigma = 1}$$

Relation between MR, AR and |ep|:

$$TR = P \cdot Q \quad \Rightarrow AR = P$$

$$MR = \frac{dTR}{dQ} = P + Q \frac{dP}{dQ}$$

$$MR = P \left[1 + \frac{Q}{P} \cdot \frac{dP}{dQ} \right]$$

$\Rightarrow MR = P\left[1 + \frac{Q}{P}\cdot\frac{dP}{dQ}\right]$

$\Rightarrow MR = AR\left[1 - \frac{1}{-\frac{P}{Q}\cdot\frac{dQ}{dP}}\right]$

$\Rightarrow \overset{20}{MR} = \overset{60}{AR}\left[1 - \frac{1}{|e_p|_?}\right]$

<u>Soln</u>:- $TR = 100Q - 2Q^2 \Rightarrow AR = P = 100 - 2Q$ — (2)

$MR = \frac{dTR}{dQ} = 100 - 4Q$ — (8)

$Q = 20$ in eq (2)

$AR = 100 - 2\times20$

$AR = 60$

At $MR = 20$

$\Rightarrow 100 - 4Q = 20$

$\Rightarrow 4Q = 80$

$\boxed{Q = 20}$ — (9)

$P = 100 - 2Q$

$2Q = 100 - P$

$Q = 50 - P/2$

$|e_p| = \frac{\partial Q}{\partial P} \times \frac{P}{Q} = -1/2 \times \frac{\overset{3}{\cancel{60}}}{\cancel{20}}$

$= -3/2$

$= -1.5$