# Integral Calculus

Q. All possible values of 'a' for which $\int_1^2 a^2 + (4-4a)x + 4x^3 \, dx \leq 12$ is given by:

(a) $a = 3$ (b) $a \leq 4$ (c) $0 \leq a \leq 3$ (d) None

$$\int_1^2 a^2 + (4-4a)x + 4x^3 \, dx = a^2 - 6a + 21.$$

$$a^2 - 6a + 21 \leq 12 \Rightarrow a^2 - 6a + 9 \leq 0.$$

$$\Rightarrow (a-3)^2 \leq 0$$

As $(a-3)^2 < 0$ is not possible $\forall a \Rightarrow (a-3)^2 = 0 \Rightarrow a = 3.$

Q. Let $\frac{d}{dx} f(x) = \frac{e^{\sin x}}{x}$, $x > 0$. If $\int_1^4 \frac{2e^{\sin x^2}}{x} dx = f(k) - f(1)$

then one possible value of $k$ is:

(a) $-4$ (b) $0$ (c) $2$ (d) $16$.

$$f(x) = \int \frac{e^{\sin x}}{x} dx.$$

LHS: $\int_1^4 \frac{2e^{\sin x^2}}{x} dx.$ Let $x^2 = z.$ , $x = 1, z = 1$

$2x\,dx = dz$ $x = 4, z = 16.$

$dx = \frac{dz}{2\sqrt{z}}.$

$$\int_1^{16} \frac{\cancel{2} e^{\sin z}}{\sqrt{z}} \cdot \frac{dz}{\cancel{2}\sqrt{z}}$$

$f(k) - f(1)$

$$\int_1^{16} \frac{e^{\sin z}}{z} dz = \left[f(z)\right]_1^{16} = f(16) - f(1)$$

$\therefore$ Comparing: $k = 16$.

Q. The eqn: $\int_{-\pi/4}^{\pi/4} a|\sin x| + \dfrac{b\sin x}{1+\cos x} + c \, dx = 0$ gives a relation b/w:-

(a) a, b, c (c) a, b

(b) b, c ~~(d)~~ a, c.

$$\int_{-\pi/4}^{\pi/4} a|\sin x| \, dx + b\int_{-\pi/4}^{\pi/4} \frac{\sin x}{1+\cos x} dx + \int_{-\pi/4}^{\pi/4} c \, dx = 0.$$

$\to 0$. $\to$ remains.

$g(x)$, then

$g(-x) = g(x) \Rightarrow$ even fn.

$\int_{-a}^{+a} g(x)\, dx = 2\int_0^a g(x)\, dx$ if $g(x)$ is even

$f(x)$, $f(-x) = -f(x) \Rightarrow$ odd fn.

$\int_{-a}^{+a} f(x)\, dx = 0$, if $f(x)$ is odd fn.

$$= 2\int_0^{\pi/4} a \sin x \, dx + \int_{-\pi/4}^{\pi/4} c \, dx = 0 \Rightarrow \text{Relation b/w } a \ \& \ c$$

$$f(x) = \frac{\sin x}{1+\cos x}$$

$$f(-x) = \frac{\sin(-x)}{1+\cos(-x)} = -\frac{\sin x}{1+\cos x} = -f(x) \Rightarrow \text{odd fn.}$$

Q. Let $a \in [0, 1]$ and $I(a) = \int_{a-1}^{a+1} e^{-|x|} dx$. Then the value of 'a' that maximizes $I(a)$ is:

(a) 0 (b) 1 (c) $1/2$ (d) None.

(b) 1 (c) $\frac{1}{2}$ (d) None.

$0 \leq a \leq 1$

$-1 \leq (a-1) \leq 0 \Rightarrow$ -ve range

$1 \leq (a+1) \leq 2 \Rightarrow$ +ve range.

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$I(a) = \int_{a-1}^{a+1} e^{-|x|}\,dx = \underbrace{\int_{a-1}^{a} e^{-|x|}\,dx}_{I_1} + \underbrace{\int_{a}^{a+1} e^{-|x|}\,dx}_{I_2}$$

+ve, -ve, +ve, (+ve)

$$I_2 = \int_a^{a+1} e^{-x}\,dx$$

$$I_1 = \int_{a-1}^{0} e^{-|x|}\,dx + \int_0^a e^{-|x|}\,dx = \int_{a-1}^{0} e^{x}\,dx + \int_0^a e^{-x}\,dx.$$

$$I(a) = I_1 + I_2 = \int_{a-1}^{0} e^{x}\,dx + \int_0^a e^{-x}\,dx + \int_a^{a+1} e^{-x}\,dx.$$

HW.

For max $I'(a) = 0 \Rightarrow$ solve for 'a'.