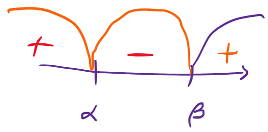
Q. Consider the diff eqn: $\frac{dy}{dx} = y(y-1)(y-2)$. Which of the following is correct?

(a) If $y(0) = 0.5$, y is decreasing

(b) If $y(0) = 1.2$, y is increasing

~~(c)~~ If $y(0) = 2.5$, y is unbounded

(d) If $y(0) < 0$, y is bounded below.



$\Rightarrow (x-\alpha)(x-\beta)$

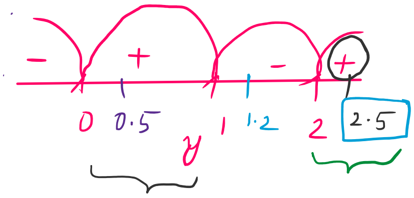
$(x-\alpha)(x-\beta) = 0 \Rightarrow x = \alpha, \beta$.

$$\frac{dy}{dx} = y(y-1)(y-2)$$

$$\frac{dy}{dx} = 0 \Rightarrow y(y-1)(y-2) = 0 \Rightarrow y = 0, 1, 2.$$

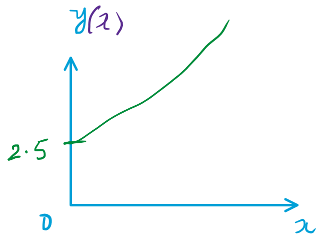
$$\frac{dy}{dx} > 0 \Rightarrow y \in (0,1) \cup (2, \infty)$$

$$\frac{dy}{dx} < 0 \Rightarrow y \in (-\infty, 0) \cup (1, 2)$$



$y = \frac{1}{x^2}$

'0' → lower bound



Q. If an integral curve (soln of the diff eqn) of the differential eqn $(y-x)\frac{dy}{dx} = 1$ passes through $(0,0)$ and $(\alpha, 1)$, then $\alpha = ?$

(a) $2 - \frac{1}{e}$ (b) $1 - \frac{1}{e}$ ~~(c)~~ $\frac{1}{e}$ (d) $1 + e$.

$$(y-x)\frac{dy}{dx} = 1. \Rightarrow \frac{dx}{dy} + x = y$$

→ Linear Diff Eqn

$$\frac{dx}{dy} + P(y)\cdot x = Q(y).$$

$$I.F = e^{\int P \cdot dy}$$

$$I.F = e^{\int 1 \cdot dy} = e^{y}.$$

$$e^{y}\left[\frac{dx}{dy} + x\right] = e^{y}\cdot y$$

$$\therefore \frac{d}{dy}\left[x \cdot e^{y}\right] = e^{y}\cdot y$$

Int: $\int d\left[x \cdot e^{y}\right] = \int y \cdot e^{y}\, dy$

$$x e^{y} = y e^{y} - \int e^{y}\, dy$$

$$\boxed{x e^{y} = y e^{y} - e^{y} + c} \to \text{Integral curve}$$

$$x = y - 1 + c \cdot e^{-y} \quad \text{---} \quad (i)$$

(i) passes through $(0,0) \Rightarrow 0 = 0 - 1 + c \cdot e^{0} \Rightarrow c = 1.$

$$x = y - 1 + e^{-y}.$$

(i) passes through $(\alpha, 1)$

$$\alpha = \cancel{1} - \cancel{1} + e^{-1} \Rightarrow \alpha = e^{-1} = 1/e.$$

Q. Given that, there is a common solution to the following diff eqns: $P: y' + 2y = e^{x} y^{2},\ y(0) = 1$

$Q: y'' + 2y' + \alpha y = 0.$

Find α. Hence find soln of Q.

$P: \quad y' + 2y = e^{x} y^{2}, \quad y(0) = 1$

$$\frac{dy}{dx} + 2 \cdot y = e^{x} y^{2}.$$

$$y^{-2} \frac{dy}{dx} + 2 \cdot y^{-1} = e^{x} \Rightarrow \text{Bernoulli Eqn.}$$

Let $y^{-1} = v$

Diff: $(-1) y^{-2} \cdot \frac{dy}{dx} = \frac{dv}{dx} \Rightarrow y^{-2} \cdot \frac{dy}{dx} = -\frac{dv}{dx}.$

Replacing: $-\frac{dv}{dx} + 2v = e^{x}.$

$$\frac{dv}{dx} - 2v = -e^{x} \quad [\text{Linear Diff Eqn in } v, x]$$

$$I.F = e^{\int -2\, dx} = e^{-2x}.$$

Diff Eqn × I.F : $\frac{d}{dx}\left[v \cdot e^{-2x}\right] = -e^{x} \cdot e^{-2x}$

Int: $\int d\left[v \cdot e^{-2x}\right] = -\int e^{-x}\,dx$

$$v \cdot e^{-2x} = e^{-x} + c.$$

$$\Rightarrow \frac{e^{-2x}}{y} = e^{-x} + c.$$

As $y(0) = 1 \Rightarrow \frac{e^{0}}{1} = e^{0} + c \Rightarrow c = 0.$

Soln: $\frac{e^{-2x}}{y} = e^{-x} \Rightarrow y = \frac{e^{-2x}}{e^{-x}} = \frac{e^{x}}{e^{2x}} = e^{-x}.$

$\boxed{\text{Soln of } P: \ y = e^{-x}}$ = Soln of Q.

Q: $y'' + 2y' + \alpha y = 0$, soln: $y = e^{-x}$.

$y' = -e^{-x}$

$y'' = e^{-x}$.

Replacing the soln:

$$e^{-x} + 2(-e^{-x}) + \alpha e^{-x} = 0.$$

$$-e^{-x} + \alpha e^{-x} = 0$$

$$\alpha e^{-x} = e^{-x} \Rightarrow \boxed{\alpha = 1}.$$

Q: $y'' + 2y' + y = 0.$

Let e^{mx} be a trial soln.

AE : $m^2 + 2m + 1 = 0 \Rightarrow (m+1)^2 = 0 \Rightarrow \underbrace{m = -1, -1}$

↳ Real & Equal Roots.

$$y = (c_1 + c_2 x)\, e^{-x}$$

HW.

Q. Let $y(x)$ be a solution to the differential eqn $\frac{d^2y}{dx^2} - y = 0$

s.t $y(0) = 2$ and $y'(0) = 2\alpha$ Find all

Q. Let $y(x)$ be a solution to the differential eqn $\dfrac{d^2y}{dx^2} - y = 0$ s.t $y(0) = 2$ and $y'(0) = 2\alpha$. Find all values of $\alpha \in [0, 1)$ s.t infimum of the set $\{y(x) \mid x \in \mathbb{R}\}$ is ≥ 1.