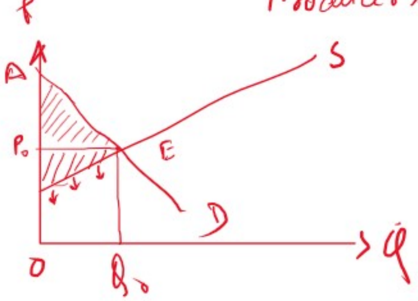
12 March 2024 09:28 PM

Application of Integration



Producer's surplus = $\underset{\text{equil price \& quant}}{OP_0 \times OQ_0} - \int_0^{Q_0} S(Q)\, dQ$

Consumer's surplus = $\int_0^{Q_0} D(Q)\, dQ - OP_0 \times OQ_0$

Q1. The marginal revenue fn of a competitive firm is given as $MR = 15 - 3x^2$.
What would be the demand curve of the firm in the market. AR ⇒ TR

$$TR = \int (MR)\, dx = \int (15 - 3x^2)\, dx$$
$$= \int 15\, dx - 3\int x^2\, dx$$
$$= 15x - \cancel{3}\frac{x^3}{\cancel{3}} + \cancel{I}C$$

$$TR = 15x - x^3 + \cancel{I}C$$

If $x = 0 \Rightarrow TR = 0$, then $0 = 15(0) - 0^3 + IC$
$$IC = 0$$

$$\therefore TR = 15x - x^3$$
$$\therefore AR = \frac{TR}{x} = 15 - x^2 \text{ (ans)}.$$

Q2. The marginal cost of x shirts is $10x - 30x^2$.
The total cost of producing a pair of shirt is Rs 40. Obtain the total and average cost function.

function.

$$MC = 10x - 30x^2$$

$$\therefore TC = \int MC\,dx = \int (10x - 30x^2)\,dx$$

$$TC = \frac{\cancel{10}^{5} x^2}{\cancel{2}} - \frac{\cancel{30}^{10} x^3}{\cancel{3}} + IC$$

$$TC = 5x^2 - 10x^3 + IC$$

at $x = 2 \Rightarrow TC = 40$

$$\therefore 40 = 5(2)^2 - 10(2)^3 + IC$$

or, $40 = 20 - 80 + IC$

or, $40 - 20 + 80 = IC$

or $\boxed{IC = 100}$

$\therefore TC = 5x^2 - 10x^3 + 100$ (ans).

$AC = \frac{TC}{x} = 5x - 10x^2 + \frac{100}{x}$

(ans)

3. The marginal cost of a company is given by

$$MC = 75 + 20x - 3x^2$$

If the fixed cost is Rs 1000, how much cost is required to produce 10 units?

$x = 10$

$IC = 1000$

$TC = ?$

$$MC = 75 + 20x - 3x^2$$

$$TC = \int MC\,dx = \int 75\,dx + 20\int x\,dx - 3\int x^2\,dx$$

$$= 75x + 20\frac{x^2}{2} - 3\frac{x^3}{3} + IC$$

$$TC = 75x + 10x^2 - x^3 + IC.$$

Since at $x = 10$ $FC = IC = 1000$

$$\therefore TC = 75(10) + 10(10)^2 - (10)^3 + 1000$$

$= 750 + 1000 - \cancel{1000}$ $\cancel{+1000}$

$\boxed{TC = 1750 (\text{ans})}$.

Q. Let the demand function be given as $P_d = (10-q)$ $f(q)$ supply function be given as $P_s = q+2$.

Find consumer surplus and producer's surplus.

In equil : $P_d = P_s$

$$10 - q = q + 2$$

$$8 = 2q$$

$$\boxed{q = 4}$$

$\therefore P = 4+2 = 6$

$$CS = \int_0^q D(q)\, dq - P \times Q$$

$$= \int_0^4 (10-q)\, dq - 24$$

$$= \left[10\int_0^4 dq - \int_0^4 q\, dq\right] - 24$$

$$= 10[q]_0^4 + \left[\frac{q^2}{2}\right]_0^4 - 24$$

$$= 10(4-0) + \frac{4^2 - 0^2}{2} - 24$$

$$= 40 + 8 - 24$$

$$= 48 - 24$$

$P = f(q)$
$P = D(q)$
$P_d = (10-q)$

$PS = P \times Q - \int_0^q S(q)\, dq$

$PS = 24 - \int_0^4 (q+2)\, dq$

$= 48 - 24$

$= 24$ (ans).

$PS = 24 - \int_0^4 (q+2)\,dq$

$= 24 - \left[\frac{q^2}{2}\right]_0^4 - 2[q]_0^4$

$= 24 - \left(\frac{16-0}{2}\right) - 2(4-0)$

$= 24 - 8 - 8$

$= 24 - 16$

$= 8$

Q. The supply function of a firm is given by $p = q^2 + 2q + 1$. Given that the equilibrium is reached at $q = 3$, find producer's surplus at the equilibrium quantity.

$q = 3 \quad \therefore p = 3^2 + 2\times 3 + 1$

$= 9 + 6 + 1$

$= 16$

$$PS = \int_0^q S(q)\,dq - P \times q$$

$$\int_0^3 (q^2 + 2q + 1)\,dq - (16 \times 3)$$

⋮

TRY YOURSELF.

Change in capital stock w.r.t. time

$\frac{dK}{dt} = I$ (investment)

$\boxed{K = \int I\,dt}$

Q. The investment function of a firm is given by

5 constant

I.

3. The investment function of a firm is given by

$$I(t) = 4bt \cdot w$$

Find the capital stock. K

b constant

I inv

t time

K capital stock

$$K = \int I(t)\,dt = \int (4bt)\,dt = 4b\frac{t^2}{2} + I_c$$

$$K(t) = 2bt^2 + IC \qquad K = \frac{\cancel{4}^{2}bt^2}{\cancel{2}} + IC$$

at $t = 0$

$$K(0) = 2b(0)^2 + IC$$

$\therefore IC = K(0)$

$t=0 \quad K(t) = 2bt^2 + K(0)$ (ans)