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Joint Probability Distribution

↓

two r.v X and Y (pairs)

$P(X=x)$ $\qquad$ $P(Y=y)$

Joint occurrence $\qquad P_{ij}$

x $\qquad$ y

x \ y	y_1 1	y_2 2	. . .	y_m 3	Marginal total of X
x_1 1	P_{11}	P_{12}	. . .	P_{1m}	P_{10}
x_2 2	P_{21}	P_{22}	. . .	P_{2m}	P_{20}
⋮	⋮				⋮
x_n 3	P_{n1}	P_{n2}	. . .	P_{nm}	P_{n0}
Marginal total of Y	P_{01}	P_{02}	. . .	P_{0m}	1

sum of total probabilities

sum of total probability

(Marginal Distribution of X)

r.v (X)	P_{i0} ie $P(X=x)$
x_1	P_{10}
x_2	P_{20}
x_3	P_{30}
⋮	⋮

x_3	P_{30}
$\vdots$	$\vdots$
x_m	P_{m0}

$\Sigma p = 1$

Conditional probability distribution of x when $y = y_1$

r.v (x)	Conditional prob $X / Y = y_1$ or $P(X \mid Y = y_1)$
x_1	P_{11} / P_{01}
x_2	P_{21} / P_{01}
x_3	P_{31} / P_{01}
$\vdots$	$\vdots$
x_m	P_{m1} / P_{01}

$E(x) = \Sigma x . p$

$V(x) = E(x^2) - E(x)^2$

Two variables $\rightarrow$ X, Y

$V(x+y)^2$

$x^2 + y^2 + 2xy$

cov

$x^2 + y^2 - 2xy$

$V(X+Y) = V(X) + V(Y) + 2\,cov(X,Y)$

$V(X-Y) = V(X) + V(Y) - 2\,cov(X,Y)$

What is the formula for cov (x, y)

$cov(X,Y) = E(XY) - E(X)E(Y)$

Always remember if x and y are two independent random variables, then $Cov(x,y) = 0$.

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$\therefore V(X+Y) = V(X-Y) = V(X) + V(Y)$.

Q> X and Y are two random variables having joint probability distribution as given below:

X \ Y	0	1	2	Marginal total of X
0	0.1	0.1	0.1	0.3
1	0.1	0.2	0.1	0.4
2	0.1	0.1	0.1	0.3
Marginal total of Y	0.3	0.4	0.3	1.0

Write down the marginal distribution of X and the conditional distribution of Y, given X=1.

Also find mean/expection (μ_x or $E(x)$)

μ_Y, σ_X, σ_Y, correlation coeff X, Y

s.d x, s.d y

$= \sqrt{\ }$

$P(X=2 \mid Y=0)$

$P(X+Y \geq 3)$

(i) Marginal Distribution of X

r.v (x)	P (X=x) or (marginal total
0	0.3
1	0.4
2	0.3
Total	$\Sigma P = 1$

Total	20	

(ii) conditional probability of Y given X = 1

r.v (value of Y)	P(X | Y=1)
0	$0.1/0.4 = 0.25$
1	$0.2/0.4 = 0.5$
2	$0.1/0.4 = 0.25$

(iii) $\mu_x = E(x) = \sum x \cdot P(x=x)$

$= 0 \times 0.3 + 1 \times 0.4 + 2 \times 0.3$

$= 1.$

(iv) $\mu_y = E(y) = \sum y \cdot P(y=y)$

$= 0 \times 0.3 + 1 \times 0.4 + 2 \times 0.3 = 1.$

(v) $\sigma_x = \sqrt{V(x)}$

$V(x) = E(x^2) - E(x)^2$

Now $E(x^2) = \sum x^2 \cdot P(x=x)$

$= 0^2 \times 0.3 + 1^2 \times 0.4 + 2^2 \times 0.3$

$= 1.6$

$= \quad 1.6$

$\therefore V(X) = 1.6 - 1^2 = 1.6 - 1 = 0.6$

$\sigma_x = \sqrt{V(X)} = \sqrt{0.6} = 0.774$ (ans)

(vi) $\sigma_y = \sqrt{V(Y)}$

$V(Y) = E(Y^2) - E(Y)^2$

Now $E(Y^2) = \sum y^2 P(Y = y)$

$= \quad 1.6$

$V(Y) = 0.6 \qquad \therefore \sigma_y = \sqrt{V(Y)} = 0.774$ (ans).

(vii) coeffecient of corelation X and Y.

$$r_{x,y} = \frac{\operatorname{cov}(X,Y)}{\sigma_x \sigma_y}$$

Now $\operatorname{cov}(X,Y) = \underset{\sim}{E(XY)} - E(X)E(Y)$

$E(XY) = \sum (XY) P_{ij}$

$= \quad 1$

$\therefore \operatorname{cov}(X,Y) = E(XY) - E(X)E(Y)$

$1 - 1$

$= 0$

$\therefore r_{x,y} = 0$

$\therefore r_{x,y} = 0$

$P(X=2/Y=0) = \dfrac{P(X=2 \cap Y=0)}{P(Y=0)} = \dfrac{0.1}{0.3} = 1/3$ 0.66 (ans)

Conditional probability of X and Y

$$\boxed{P(X/Y) = \frac{P(X \cap Y)}{P(Y)}}$$

$P(X+Y \geq 3) = P(X=1, Y=2) + P(X=2, Y=1) + P(X=2, Y=2)$

$= 0.1 + 0.1 + 0.1 = 0.3$ (ans)

Q2 For two random variables X and Y

$E(X) = 8 \quad E(Y) = 6$

$Var(X) = 16 \quad Var(Y) = 36$

$r_{x,y} = 0.5$

find (a) $E(XY)$

(b) $Cov(X, X+Y)$ (c) $Var(2X - 5Y)$

(a) given $E(X) = 8$ $E(Y) = 6$

$V(X) = 16 \rightarrow \sigma_x = 4$

$V(Y) = 36 \rightarrow \sigma_y = 6$

0.5

$V(Y) = 36 \Rightarrow \sigma_Y = 6$

$r_{X,Y} = 0.5$

$\Rightarrow \dfrac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y} = 0.5$

$\Rightarrow \operatorname{cov}(X,Y) = \boxed{0.5 \times \sigma_X \times \sigma_Y}$

$\Rightarrow E(XY) - E(X)E(Y) = 0.5 \times 4 \times 6$

$\Rightarrow E(XY) - 8 \times 6 = 0.5 \times 4 \times 6$

$\Rightarrow E(XY) = 0.5 \times 24 + 48$

$= 12.0 + 48$

(a) $E(XY) = 60$ (ans)

(b) $\operatorname{cov}(X, X+Y) = \operatorname{var}(X) + \operatorname{cov}(X,Y)$

$= 16 + 12$

$= 28$ (ans)

(c) $\operatorname{Var}(2X - 5Y) = 4V(X) + 25V(Y) - 20\operatorname{cov}(X,Y)$

$= (4 \times 16) + (25 \times 36) - (20 \times 12)$

$=$ (ans)