Q. A representative firm in a competitive mkt is having cost fn: $c_i(q_i) = 0.1\,q_i^2 + q_i + 10$ and there are 100 firms in the mkt.

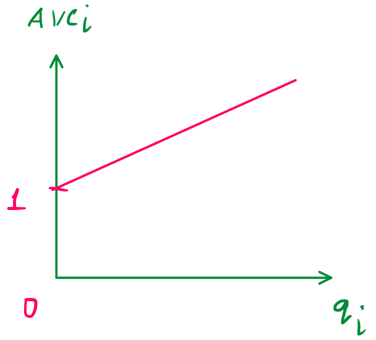
(i) Find the firm's supply curve & mkt supply curve.

(ii) If the mkt demand curve is $D = -400P + 4000$. Find the equilibrium price in the mkt.

$$c_i(q_i) = 0.1\,q_i^2 + q_i + 10.$$

$$VC_i = 0.1\,q_i^2 + q_i$$

$$AVC_i = 0.1\,q_i + 1$$ --- Linear.



For min AVC, $q = 0 \Rightarrow \min AVC_i = 1$.

(i) Firm's supply curve:

$$\pi_i = Pq_i - (0.1\,q_i^2 + q_i + 10)$$

$$\frac{\partial \pi_i}{\partial q_i} = 0 \Rightarrow P - 0.2\,q_i - 1 = 0.$$

$$\Rightarrow (P-1) = 0.2\,q_i \Rightarrow q_i = 5(P-1)$$

$\therefore$ Firm's supply curve $s(P) = \begin{cases} 5(P-1), & P > 1 \\ 0, & P \le 1 \end{cases}$

Market supply curve $S(P) = \begin{cases} 500(P-1), & P > 1 \\ 0, & P \le 1 \end{cases}$

(ii) $D = -400P + 4000$.

At equi, $D = S \Rightarrow -400P + 4000 = 500(P-1)$

$4500 = 900P \Rightarrow P^* = 5$.

$Q^* = 2000$.

(iii) If the govt imposes a per-unit tax of Rs. t on the producers, find the post-tax firm and industry supply curve & the eq. price.

New cost to the producers $= C_i(q_i) + tq_i$

$$\pi_i = Pq_i - (0.1\, q_i^2 + q_i + 10 + tq_i)$$

$$\frac{\partial \pi_i}{\partial q_i} = 0 \Rightarrow P - 0.2q_i - 1 - t = 0$$

$$(P - 1 - t) = 0.2\, q_i$$

$$q_i = 5(P - 1 - t)$$

Post-tax firm's supply curve: $P - 1 - t > 0 \Rightarrow P > (t+1)$

$$s(P) = \begin{cases} 5(P-1-t) & ; P > (t+1) \\ 0 & ; P \leq (t+1) \end{cases}$$

Post-tax mkt supply curve:

$$S(P) = \begin{cases} 500(P-1-t) & ; P > (t+1) \\ 0 & ; P \leq (t+1) \end{cases}$$

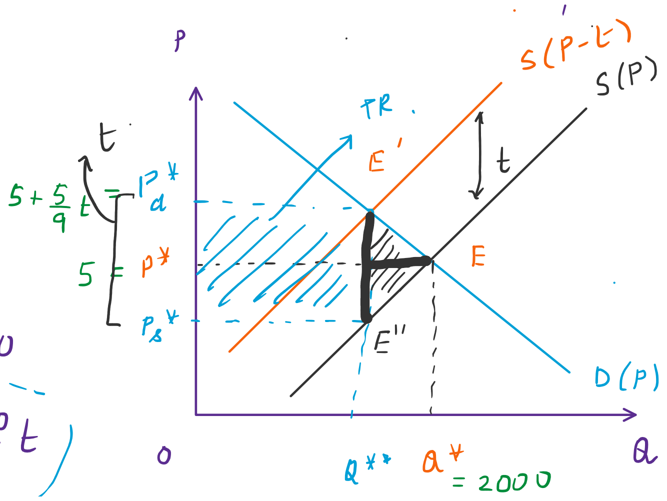
Post-tax equillibrium: $-400P + 4000 = 500(P - 1 - t)$

$$P = 5 + \frac{5}{9}t$$

[Price does not increase by the full-extent of the tax in the short run].

(iv) Calculate the DWL due to tax.

DWL = area(EE'E")



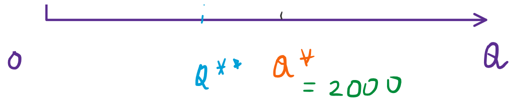
Post price price $= 5 + \frac{5}{9}t$

Post tax quantity: $D = -400P + 4000$

$$Q^{**} = 2000 - \frac{2000}{9}t$$

$$\boxed{Q^{**} = 2000 - \frac{2000}{9}t}$$

↳ Post-tax quantity



$$\underbrace{2000 - \frac{2000}{9}t}$$

$$dQ = \frac{2000}{9}t$$

$$DWL = \frac{1}{2}t \cdot dQ$$

$$= \frac{1}{2} \cdot t\left(\frac{2000}{9}t\right) = \boxed{\frac{1000}{9}t^2}$$

(v) Find the Tax Revenue maximizing rate of 't'.

Tax Revenue $= t \cdot Q$ [t = per unit rate of tax, Q = Quantity sold in the mkt]

$\therefore$ Find 't' that maximizes $TR = t \cdot Q$.

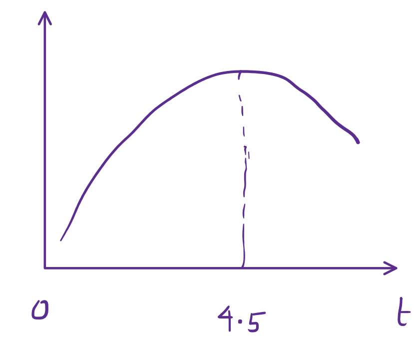
$$TR = t \cdot Q^{**} = t\left(2000 - \frac{2000}{9}t\right)$$

For max:

$$\therefore \frac{dTR}{dt} = 0 \Rightarrow 2000\left(1 - \frac{2}{9}t\right) \Rightarrow t^{*} = 4.5$$

[Idea: $t\uparrow \Rightarrow TR = (\overset{\uparrow}{t} \cdot \overset{\downarrow}{Q})\downarrow$]

Laffer curve



Note: In case the Demand & supply Curves are non-linear:-

Define: Inverse dd curve: $P_d(Q)$, $P_d' < 0$

Inverse ss curve: $P_s(Q)$, $P_s' > 0$.

$$DWL = \int_{Q^{**}}^{Q^{*}} \left[P_d(Q) - P_s(Q)\right] \cdot dQ.$$