$$TSS = ESS + RSS.$$

(II) To test: $H_0: \beta_2 = \beta_3 = \cdots = \beta_k = 0$ vs H_1: atleast one $\beta_i \neq 0, i = 2, 3, \cdots, k$

Obj: To test whether the set of explanatory variables used to model Y is actually meaningful or not.

Recall from ANOVA: $ESS = \hat{y}'\hat{y} = f(\hat{\beta}_2, \hat{\beta}_3, \ldots, \hat{\beta}_k)$

If H_0 is false $\Rightarrow$ we expect ESS to be high. [RSS to be low]

If H_0 is true $\Rightarrow$ we expect ESS to be low [RSS to be high]

& we know:-

$$\frac{\sum e_i^2}{\sigma^2} \sim \chi^2_{(n-k)} \Rightarrow \frac{e'e}{\sigma^2} \sim \chi^2_{(n-k)}$$

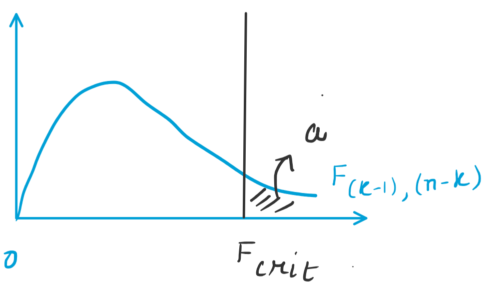
$$\frac{\sum \hat{y}_i^2}{\sigma^2} \sim \chi^2_{(k-1)} \Rightarrow \frac{\hat{y}'\hat{y}}{\sigma^2} \sim \chi^2_{(k-1)}$$

independent χ^2's.

$$\therefore \quad F = \frac{\frac{\hat{y}'\hat{y}}{\cancel{\sigma^2}}/(k-1)}{\frac{e'e}{\cancel{\sigma^2}}/(n-k)} \sim F_{(k-1),(n-k)}$$

$$F = \frac{ESS/(k-1)}{RSS/(n-k)} \sim F_{(k-1),(n-k)}$$

Test-statistic



Note: If H_0 is true, F will be low

If H_0 is false, F will be high. $\Rightarrow$ we will reject H_0 if F is high.

Testing Rule: we will reject H_0 at a% L.O.S if $F > F_{crit}$.

Q. Consider a sample data of size (n) collected from a firm which includes output level (q), Labour employed (L) & Capital (K). If the firm implements a Cobb-Douglas prodn fn: $q = A L^{\beta_2} K^{\beta_3}$

... Labour employed (L) & Capital (K).

If the firm implements a Cobb-Douglas prodn fn: $q = A L^{\beta_2} K^{\beta_3}$ then estimate A, β_2, β_3 from the data & formulate a test to check if the prodn fn satisfies CRS.

q	L	K
q_1	L_1	K_1
q_2	L_2	K_2
$\vdots$	$\vdots$	$\vdots$
q_n	L_n	K_n

$$q_i = A L_i^{\beta_2} K_i^{\beta_3}.$$

Take log: $$\underbrace{\ln q_i}_{Y_i} = \underbrace{\ln A}_{\beta_1} + \beta_2 \underbrace{\ln L_i}_{X_{2i}} + \beta_3 \underbrace{\ln K_i}_{X_{3i}}$$

True Model: $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$.

where
$Y_i = \ln q_i$
$X_{2i} = \ln L_i$
$X_{3i} = \ln K_i$
$\beta_1 = \ln A$.

As the model is linear in parameters, use OLS to estimate the model.

Estimated Model: $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i}$

$\hat{\beta}_2$: estimate of β_2

$\hat{\beta}_3$: estimate of β_3.

$\hat{\beta}_1$: estimate of $\ln A$ $\Rightarrow$ estimate of $A = e^{\hat{\beta}_1}$

To check if the prodn fn satisfies CRS, we will test: $H_0: \beta_2 + \beta_3 = 1$ vs $H_1: \beta_2 + \beta_3 \neq 1$.

$$H_0: \underbrace{\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}}_{\beta} = \underbrace{\begin{bmatrix} 1 \end{bmatrix}}_{r}.$$

$\therefore$ Rewriting: $H_0: R\beta = r$ vs $H_1: R\beta \neq r$ $\Rightarrow$ General Representation.

Eg 2: True Model: $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + u_i$

To test: H_0 : $\beta_1 - \beta_3 = 2$ vs H_1: H_0 is not true.

$\beta_2 + \beta_4 = 1$ ↪ composite hypothesis. [4 Parameter, 2 condition].

$$\underbrace{\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}}_{\substack{R \\ 2\times 4}} \underbrace{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}}_{4\times 1} = \underbrace{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}_{\substack{2\times 1 \\ r}}$$

(III) Customized Hypothesis:

Consider the k-variable model: $Y = X\beta + U$

k = Parameter
q = conditions.

Suppose, there are 'q' restrictions on the parameters that we want to test. $[q < k]$.

To test: H_0: $\underset{q\times k}{R}\ \underset{k\times 1}{\beta} = \underset{q\times 1}{r}$ vs H_1: $R\beta \neq r$.

We know: $U \sim N[\underset{\sim}{0}, \sigma^2 I]$.

and we know that $\hat{\beta}$ is linear in U.

$\therefore\ \hat{\beta} \sim N[E(\hat{\beta}), Var(\hat{\beta})]$.

$\Rightarrow\ \boxed{\hat{\beta} \sim N[\beta, \sigma^2 (X'X)^{-1}]}$.

Note: To do hypothesis testing on $R\beta$, we will use $R\hat{\beta}$ ← vector to develop the test-statistic for the test.

HW $E[R\hat{\beta}] =$

$Var[R\hat{\beta}] =$