03 August 2023 11:07

Cyclicity of Powers.

last digit	Power=1	Power=2	Power 3	Power 4	Power 5
1	1	1	1	1	1
2	2	4	8 (10−2)	6 (10−4)	2
3	3	9	7 (10−3)	1	3
4	4	6	4	6	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	9	3 (10−7)	1	7
8	8	4	2 (10−2)	6	8
9	9	1	9	1	9

$21 \times 21 = 441$

Nos which are missing in power 2 = 2, 3, 7, 8.

$\Downarrow$

No perfect square can end in 2, 3, 7, 8.

$4^{odd} \to 4$, $4^{even} \to 6$ (10−4)

$9^{odd} \to 9$, $9^{even} \to 1$ (10−9)

$2 \to 2, 4, 8, 6$

$8 \to 8, 4, 2, 6$

$3 \to 3, 9, 7, 1$

$7 \to 7, 9, 3, 1$

$\cancel{4}3\,2^{27} \to$ last digit? $\to 8$

$27 \div 4$: quotient 6, $27 - 24 = 3$ Rem

$2^{27} \equiv 2^3 = 8$

$R\left(\frac{27}{4}\right) = 3$

$2^1 = 2^5 = 2^9 \ldots$

$R\left(\frac{9}{4}\right) = 1$

$2^{32} \to 2^4 \to 6$

$723^{327} \to 3^{27} \to 3^3 \to 7$

$438^{843} \times 843^{438} \to 8^{43} \times 3^{38} \to 8^3 \times 3^2 \to 2 \times 9 \to 8$

Remainder

$R\left(\frac{2^{19}}{7}\right) = 2$

$R\left(\frac{2^9}{7}\right) = R\left[\left(\frac{2^3}{7}\right)\right]^3$

$= \left[R\left(\frac{2^3}{7}\right)\right]^3$

$2^9 = 2^{3\times 3}$

$2^3 = 8$

$512 \div 7$: quotient 73

$2^{\cdot} = 8$

$= (2^3)^3$

$2^3 = 8$

$$7 \overline{)512}$$
$$49$$
$$22$$
$$21$$
$$1$$

$= (1 \langle \tfrac{2}{7} \rangle)$

$= 1^3 = 1$

$19 = 3 \times 6 + 1$

$2^{19} = 2^{3\times6+1} = 2^{3\times6} \times 2^1$

$= (2^3)^6 \times 2^1 \rightarrow \left[R\left(\frac{2^3}{7}\right)\right]^6 \times R\left(\frac{2}{7}\right)$

$x^a \times x^b = x^{a+b}$

$(x^a)^b = x^{ab}$

$1^6 \times 2 = 1 \times 2 = 2$

$R\left(\frac{2^{19}}{9}\right) = 2.$ $\qquad R\left(\frac{2^3}{9}\right) \Rightarrow 8.$

$8 = 9 \times 0 + 8$

$= 9 \times 1 - 1$ negative rem

Actual rem = Divisor + (-ve rem)

$R\left(\frac{2^3}{9}\right) = -1$

$\left[R\,\frac{2^3}{9}\right]^6 \times \left(R\,\frac{2}{9}\right)$

$(-1)^6 \times 2 = 1 \times 2 = 2$