

Cyclicly of Powers.

last digit

$\frac{20}{4} = 5$

	Power=1	Power=2	Power=3	Power=4
1	1	1	1	1
2	2	4 ✓	8 (10-2)	6 (10-4)
3	3	9 ✓	7 (10-3)	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9 ✓	3 (10-7)	1
8	8	4 ✓	2 (10-2)	6
9	9	1	9	1

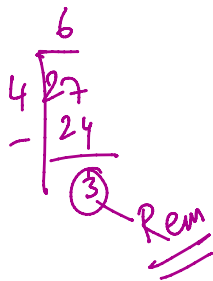
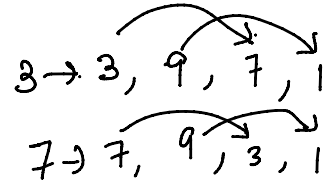
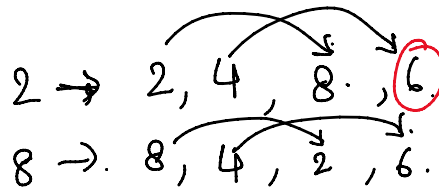
Power 5 Nos which are missing in power 2 = 2, 3, 7, 8.

↓

No perfect square can end in 2, 3, 7, 8.

4^{odd} → 4, 4^{even} → 6 (10-4)

9^{odd} → 9, 9^{even} → 1 (10-9)



27 → last digit? → 8

$2^{27} = 2^3$

$R(\frac{27}{4}) = 3$

= 8

$2^5 = 2^9$

$R(\frac{9}{4}) = 1$

$2^{32} = 2^4$ → 6

$3^{27} = 3^3$ → 7

$8^{43} \times 8^{43} = 8^{86} = 8^3 \times 8^2 = 2 \times 9$ → 8

Remainder

$R(\frac{2^{19}}{7}) = 2$

$R(\frac{2^9}{7}) = R[\frac{2^3}{7}]^3$

$= [R(\frac{2^3}{7})]^3$

$2^9 = 2^{3 \times 3}$

$= 2^{27}$

$2^3 = 8$

$7 \overline{) 512}$

$$2^3 = 8$$

$$= (2^3)^3$$

$$\frac{2^3 = 8}{1}$$

$$\begin{array}{r} 7 \overline{) 512} \\ \underline{49} \\ 22 \\ \underline{21} \\ 1 \end{array}$$

$$= \left\lfloor \left(\frac{2^3}{7} \right) \right\rfloor$$

$$= 1^3 = 1$$

$$19 = 3 \times 6 + 1$$

$$2^{19} = 2^{3 \times 6 + 1} = 2^{3 \times 6} \times 2^1$$

$$= (2^3)^6 \times 2^1 \rightarrow \left[R\left(\frac{2^3}{7}\right) \right]^6 \times R\left(\frac{2}{7}\right)$$

$$1^6 \times 2 = 1 \times 2 = 2$$

$$z^a \times z^b = z^{a+b}$$

$$(z^a)^b = z^{ab}$$

$$R\left(\frac{2^{19}}{9}\right) = 2$$

$$R\left(\frac{2^3}{9}\right) \Rightarrow 8$$

$$R\left(\frac{2^3}{9}\right) = -1$$

$$\left[R\left(\frac{2^3}{9}\right) \right]^6 \times R\left(\frac{2}{9}\right)$$

$$(-1)^6 \times 2 = 1 \times 2 = 2$$

$$8 = 9 \times 0 + 8$$

$$= 9 \times 1 - 1 \text{ (negative rem)}$$

Actual rem = Divisor + (neg rem)