w1/2 24.5 Calculate S.D.

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Q	Marks	No. of students (f)	x_i (mid value)	$x_i f_i$	x_i^2	$x_i^2 f_i$
	20-29	5	24.5			
	30-39	12	34.5			
	40-49	15	44.5			
	50-59	20	55.5			
	60-69	18	65.5			
	70-79	10	75.5			
	80-89	6	85.5			
	90-99	4	95.5			
		$N = \Sigma f_i =$		$\Sigma x_i f_i$ =		$\Sigma x_i^2 f_i$ = 323162.5

variance, $\sigma^2 = \frac{1}{N}\Sigma x_i^2 f_i - \bar{x}^2$

$\bar{x} = \frac{1}{\Sigma f}\Sigma x_i f_i$ where $N = \Sigma f_i$

$\bar{x} = \frac{1}{90} \times 5143 = 57.14$.

variance $= \frac{1}{N}\Sigma x_i^2 f_i - \bar{x}^2$

$= \left(\frac{1}{90} \times 323162.5\right) - (57.14)^2$

$= 3590.69 - 3264.97$

$= 325.72$

$sd = \sqrt{\text{variance}} = \sqrt{325.72}$

= (13.07)

Formula for combined s.d.

	Groups I	Groups II	Composite group
obs	n_1	n_2	N
mean	$\bar{x}_1$	$\bar{x}_2$	$\bar{X}$
s.d	σ_1	σ_2	σ

grouped mean $\bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{N}$$

$$\therefore N\bar{x} = n_1\bar{x}_1 + n_2\bar{x}_2$$

$\therefore$ Combined variance, $\sigma^2 = \dfrac{\Sigma n_i \sigma_i^2 + \Sigma n_i d_i^2}{N}$

where $d_i = \bar{x}_i - \bar{x}$

i.e. $d_1 = \bar{x}_1 - \bar{x}$

$d_2 = \bar{x}_2 - \bar{x}$

$d_2 = (\bar{x}_2) - \bar{x}$

∴ for two groups

$$\text{combined s.d. } \sigma = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2}{N}}$$

OR

$$\text{combined sd } \sigma = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2}{N} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{N}}$$

Q

Group of 50 boys (n_1)	Group of 40 girls (n_2)
mean score = 59.5 ($\bar{x}_1$)	mean 54.0 ($\bar{x}_2$)
sd = 8.38 (σ_1)	s.d 8.23 (σ_2)

Find the mean and sd ($\bar{x}$, σ) of the combined group of 90 children.

Given, $n_1 = 50$, $\bar{x}_1 = 59.5$, $\sigma_1 = 8.38$

$n_2 = 40$, $\bar{x}_2 = 54.0$, $\sigma_2 = 8.23$

$$\bar{X} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{(50 \times 59.5) + (40 \times 54)}{90}$$

$$= 57.055 \text{ kg}.$$

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2}{n_1 + n_2}$$

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2} + \frac{n_1 n_2}{n_1+n_2}(\bar{x}_1 - \bar{x}_2)^2$$

$$\sigma^2 = \left[\frac{50 \times 8.38^2 + 40 \times 8.23^2}{90}\right] + \left[\left(\frac{40\times 50}{90}\right)(59.5-54)^2\right]$$

CV 50 %

$\sigma = 4$

$\bar{x} = ?$

$$CV = \frac{SD}{mean} \times 100$$

$$50 = \frac{4}{\bar{x}} \times 100$$

$$\bar{x} = \frac{4}{50} \times 100^2$$

$$= 8 \quad \text{ans} \leftarrow$$

σ