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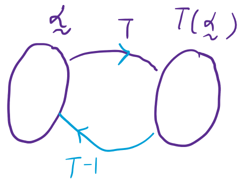


Non-singular Transformation:-

A linear transformation T is said to be non-singular if $N(T) = \{\underset{\sim}{0}\}$. [$\underset{\sim}{0}$ is also known as the trivial solution]

(*) Note: If a transformation T is non-singular, then nullity $= 0$.

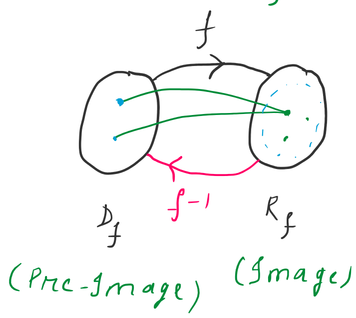
Singular Transformation:-

A transformation T is singular if $\exists$ a vector $\underset{\sim}{\alpha} \neq \underset{\sim}{0}$ s.t $T(\underset{\sim}{\alpha}) = 0$.

Digression: Fn $f(x)$: $f: D_f \to R_f$.

Injective fn: If $f(x_1) = f(x_2) \iff x_1 = x_2$; $x_1, x_2 \in D_f$.

Surjective fn (onto fn): Consider $y \in R_f$. Then $f(x)$ will be surjective if $\exists$ atleast on $\alpha \in D_f$ s.t $f(\alpha) = y$



Bijective fn: Injective + surjective.
(one-one-correspondence).

Inverse of a Fn: $f^{-1} : R_f \to D_f$ s.t $x = f^{-1}(y)$, $x \in D_f$, $y \in R_f$
[Inverse of a fn only exists if fn is bijective]

Note: [Just like fns, similar logic can be applied to linear transformation]

(i) T is non-singular iff T is one-one.

(ii) T is invertibe (T^{-1} exists) iff T is non-singular

(iii) T is inverble iff T is onto.

Non-singular $T \Rightarrow N(T) = \{\underset{\sim}{0}\}$.

$\therefore \quad T(\underset{\sim}{u}) = 0 \Rightarrow u = 0$

$T[c_1 \underset{\sim}{\alpha} + c_2 \underset{\sim}{\beta}] =$
$c_1 T(\alpha) + c_2 T(\beta)$

...gular $T \Rightarrow N(T) = \{\underset{\sim}{0}\}$.

$\boxed{T[c_1\underset{\sim}{\alpha} + c_2\underset{\sim}{\beta}] = c_1 T(\underset{\sim}{\alpha}) + c_2 T(\underset{\sim}{\beta})}$.

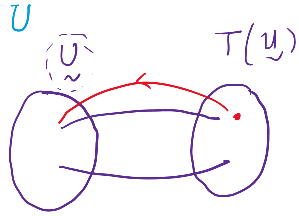
$\therefore \quad T(\underset{\sim}{u}) = 0 \Rightarrow \underset{\sim}{u} = 0$.

Injectivity $\Rightarrow T(\underset{\sim}{u_1}) = T(\underset{\sim}{u_2})$, $\underset{\sim}{u_1}, \underset{\sim}{u_2} \in U$

$\Rightarrow T(\underset{\sim}{u_1}) - T(\underset{\sim}{u_2}) = 0$.

$\Rightarrow T(\underset{\sim}{u_1} - \underset{\sim}{u_2}) = 0$.

$\Rightarrow \underset{\sim}{u_1} - \underset{\sim}{u_2} = 0 \Rightarrow \underset{\sim}{u_1} = \underset{\sim}{u_2}$.



Non-singular $\Leftrightarrow$ Injectivity $\Leftrightarrow$ T^{-1} exists.

Q. $T(x, y, z) = (x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta,\ z)$

Check for singularity of T.

$N(T) = \{(x, y, z) \mid T(x, y, z) = (0, 0, 0)\}$.

$T(x, y, z) = (0, 0, 0)$

$(x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta,\ z) = (0, 0, 0)$

$$\Rightarrow \left.\begin{array}{l} x\cos\theta - y\sin\theta = 0 \\ x\sin\theta + y\cos\theta = 0 \\ z = 0 \end{array}\right\} \Rightarrow x = 0,\ y = 0,\ z = 0.$$

$N(T) = \{\underset{\sim}{0}\} \Rightarrow$ Non-singularity.

Q. Consider $T(x, y) = (\alpha x + \beta y, \gamma x + \delta y)$. Determine the condition under which T^{-1} exist.

$T(x, y) = \underset{\sim}{0}$.

$(\alpha x + \beta y, \gamma x + \delta y) = (0, 0)$

$$\left\{\begin{array}{l} \alpha x + \beta y = 0 \\ \gamma x + \delta y = 0 \end{array}\right\} \Rightarrow \text{Solving} \qquad \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} \neq 0.$$

$\hookrightarrow$ Homogeneous system

$\alpha\delta - \beta\gamma \neq 0 \Rightarrow x = y = 0$

$\alpha\delta \neq \beta\gamma \Rightarrow T$ is non-singular

$\Rightarrow T^{-1}$ exists.

Q. $T: \mathbb{R}^3 \to \mathbb{R}^2$ s.t $T(x, y, z) = (x, y)$. Check if its one-one.

Q. $T: \mathbb{R}^3 \to \mathbb{R}^2$ s.t $T(x, y, z) = (x-y, y-z)$. Check if its one-one.

$N(T) = \underset{\sim}{0} \Rightarrow (x-y, y-z) = (0, 0)$

$$\left.\begin{aligned} x-y &= 0 \\ y-z &= 0 \end{aligned}\right\} \Rightarrow x = y = z = k \text{ (say)}.$$

$N(T) = \{(k, k, k)\} = \{k(1, 1, 1)\} \Rightarrow$ singular.

Q. $T: \mathbb{R}^3 \to \mathbb{R}^2$ s.t $T(x, y, z) = (x-y, y)$ $\Rightarrow$ Not one-one.