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$$\lim_{x\to 0} \boxed{(1+x)^{\frac{1}{x}}} = \lim_{x\to 0} y. \quad x=0. \quad (1+0)^{\frac{1}{0}}, \quad (1+0)^{\infty} = 1 \times$$

Step 1.

$$y = (1+x)^{1/x}.$$

$$\log_e y = \log_e\left[(1+x)^{1/x}\right] = \frac{1}{x}\log_e(1+x).$$

$$\lim_{x\to 0} \log_e y = \lim_{x\to 0} \frac{1}{x}\log_e(1+x)$$

Step 2

Limit of log = log of limit.

$$\log_e\left(\lim_{x\to 0} y\right) = \lim_{x\to 0} \frac{\log_e(1+x)}{x}.$$

$$x=0. \quad \frac{\log(1+0)}{0} = \boxed{\frac{0}{0}} \quad \boxed{\text{or } \frac{\infty}{\infty}}$$

L Hospital's Rule

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

where $f(a) = 0, \infty.$ $g(a) = 0, \infty.$

Step 3. apply L-Hospital's Rule.

$$\log_e\left(\lim_{x\to 0} y\right) = \lim_{x\to 0} \frac{\frac{d}{dx}\left[\log_e(1+x)\right]}{\frac{d}{dx}(x)}$$

$$\frac{d}{dx}\log_e x = \frac{1}{x}.$$

$$\frac{d}{dx}\log_e \underbrace{(1+x)}_{z}$$

$$= \frac{d}{dx}(\log z) = \frac{d\log z}{dz}\cdot\frac{dz}{dx}.$$

$$= \frac{1}{z}\cdot 1 = \frac{1}{z} = \frac{1}{1+x}.$$

$$= \lim_{x\to 0} \frac{\frac{1}{1+x}}{1}$$

$$\log_e\left(\lim_{x\to 0} y\right) = 1$$

$$\lim_{x\to 0} y = e^1 = e.$$

Ans. $\boxed{\lim_{x\to 0} (1+x)^{1/x} = e.}$

$$\lim_{x\to 1} \left(\frac{x}{x-1}\right)^{\frac{1}{x-1}} = \boxed{\lim_{x\to 1} y.} \quad \left(\frac{1}{0}\right)^{\infty}.$$

Step 1. let $y = \left(\frac{x}{x-1}\right)^{\frac{1}{x-1}}$

$$\log y = \frac{1}{x-1}\log\left(\frac{x}{x-1}\right) = \frac{1}{x-1}\left[\log x - \log(x-1)\right].$$

Step 2

$$\lim_{x\to 1} \log y = \lim_{x\to 1} \frac{\log(x) - \log(x-1)}{(x-1)}$$

$$\log\left(\lim_{x\to1} y\right) = \lim_{x\to1} \frac{\log x - \log(x-1)}{x-1}$$

step3. apply L Hospital

$$\log\left(\lim_{x\to1} y\right) = \lim_{x\to1} \frac{\frac{1}{x} - \frac{1}{x-1}}{1} = \lim_{x\to1} \frac{x-1-x}{x(x-1)}$$

$$\log_e\left(\lim_{x\to1} y\right) = \lim_{x\to1} \frac{(-1)}{x(x-1)} = -\frac{1}{0} = -\infty.$$

$$\boxed{\lim_{x\to1} y = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0.}$$

$$\lim_{x\to0} \frac{\sin x}{x} \qquad x=0 \qquad \frac{0}{0}.$$

$$\lim_{x\to0} \frac{\frac{d}{dx}(\sin x)}{\frac{d}{dx}(x)} = \lim_{x\to0} \frac{\cos x}{1} = 1$$

(a) $\lim_{x\to0} \frac{\sin x}{x}$

(b) $\lim_{t\to1} \frac{5t^4 - 4t^2 - 1}{10 - t - 9t^3}$

(c) $\lim_{x\to\infty} \frac{e^x}{x^2}$ $\quad \frac{\infty}{\infty}$

$$\lim_{t\to1} \frac{5t^4 - 4t^2 - 1}{10 - t - 9t^3} \qquad t=1 \qquad \frac{0}{0}$$

$$= \lim_{t\to1} \frac{5 \cdot 4t^3 - 4 \cdot 2t}{-1 - 9 \cdot 3t^2}$$

$$= \lim_{t\to1} \frac{20t^3 - 8t}{-1 - 27t^2} = \frac{20-8}{-1-27} = \frac{12}{-28} = -\frac{3}{7}.$$

$$\lim_{x\to\infty} \frac{\frac{d}{dx}(e^x)}{\frac{d}{dx}(x^2)}$$

$$= \lim_{x\to\infty} \frac{e^x}{2x} \qquad \frac{\infty}{\infty}.$$

$$= \lim_{x\to\infty} \frac{\frac{d}{dx}(e^x)}{2\frac{d}{dx}(x)} = \lim_{x\to\infty} \frac{e^x}{2} = \infty.$$

$$\lim_{x\to0} (x^2+x+1)^{\frac{1}{x(x+1)}} \qquad (1+0)^{\frac{1}{0}}.$$

$$= \lim_{x\to0} [1 + \underbrace{\{x(x+1)\}}_{y}]^{\frac{1}{x(x+1)}}$$

let $x(x+1) = y$

when $x\to0$, $y\to0$

$= \lim_{x\to 0} [1+2 \cdots]$

$y$

when $x \to 0$, $y \to 0$

$= \lim_{y\to 0} [1+y]^{\frac{1}{y}} = e.$

$\lim_{x\to 1} [x^2-5x+5]^{\frac{1}{x^2-5x+4}}$

$\lim_{x\to 0} (1+x)^{\frac{1}{x}}.$

$= \lim_{x\to 1} [1+(x^2-5x+4)]^{\frac{1}{x^2-5x+4}}$

$x \to 1 \quad x^2-5x+4 \to 0$

$= e.$

$\lim_{x\to\infty} \boxed{(1+e^x)^{\sin x}} = \lim_{x\to\infty} y.$

$y = (1+e^x)^{\sin x}.$

whenever $f(x)$ is in the power apply log.

$\log y = \sin x \log(1+e^x)$

$\lim_{x\to\infty} \log y = \lim_{x\to\infty} \sin x \log(1+e^x)$

$\pm 1 \qquad \infty.$

$= \lim_{x\to\infty} \frac{\log(1+e^x)}{\frac{1}{\sin x}}$ $\cancel{\frac{\infty}{\infty}}$

$= \lim_{x\to\infty} \cancel{\frac{\frac{1}{1+e^x} e^x}{-\cot x\,\mathrm{cosec}\,x}}$

$\frac{1}{\sin x} = \mathrm{cosec}\,x.$

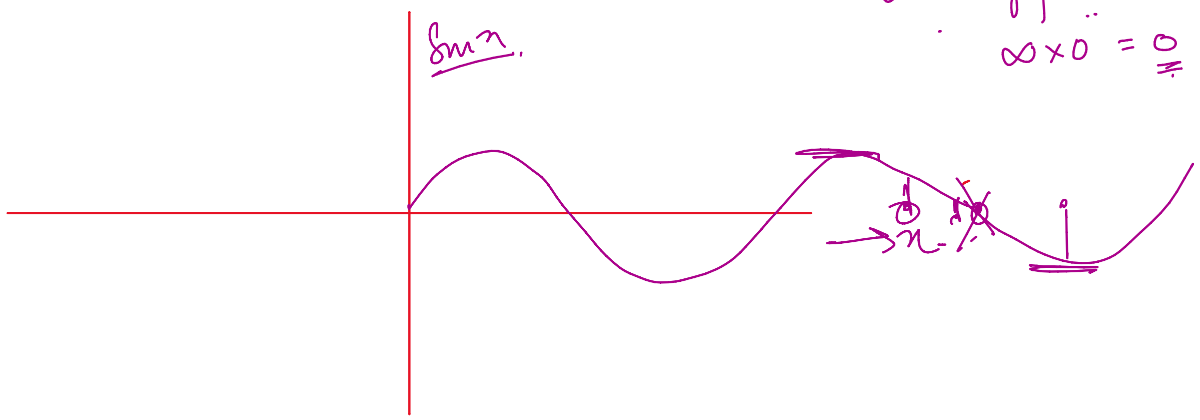
$\frac{d}{dx}\mathrm{cosec}\,x = -\mathrm{cosec}\,x\cot x.$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v-v'u}{v^2}$ $\quad \frac{d}{dx}\left(\frac{1}{\sin x}\right) = -\frac{\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}$

$= -\cot x \cdot \mathrm{cosec}\,x.$

$= \infty \times$ any finite value $= \infty$

$\infty \times 0 = 0$




$\lim_{x\to 0} (1+e^x)^{\sin x}$ $\quad x=0 \quad 2^0 = 1$

$\lim \boxed{(1+\sin x)^{e^{\frac{1}{x}}}}$ $\quad (1+0)^{e^{\infty}} = (1+0)^{\infty}.$

$$\boxed{\lim_{x\to 0}(1+\sin x)^{e^{\frac{1}{x}}}} \qquad (1+0)^{e^{\infty}} = (1+0)^{\infty}.$$

$y.$

$$y = (1+\sin x)^{e^{\frac{1}{x}}}.$$

$$\log y = e^{1/x}\log(1+\sin x)$$

$$e^{-1/0} = e^{-\infty} = \frac{1}{e^{\infty}} = 0$$

$$\lim_{x\to 0}\log y = \lim_{x\to 0} e^{1/x}\log(1+\sin x) = \lim_{x\to 0}\cancel{\frac{\log(1+\sin x)}{\frac{1}{e^{1/x}}}} \qquad \frac{0}{0}$$

$$\underline{\infty \times 0 = 0}$$

$$\frac{d}{dx}\left(\frac{-1}{x}\right) = -(-1)x^{-2}.$$

$$= \lim_{x\to 0}\frac{\log(1+\sin x)}{e^{-1/x}} = \lim_{x\to 0}\frac{\left(\frac{1}{1+\sin x}\right)\cos x}{e^{-1/x}\cdot x^{-2}}$$

$$= \lim_{x\to 0}\frac{x^2}{e^{-1/x}}\cdot\frac{(\cos x)}{(1+\sin x)}$$

$$\log_e \lim_{x\to 0} y = 0.$$

$$\boxed{\lim_{x\to 0} y = 1}$$