13 January 2024 12:50

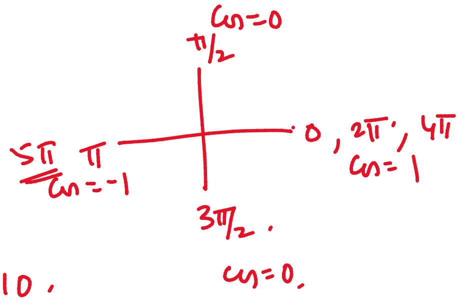
ILLUSTRATION 1 *If* $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, *where* $[\cdot]$ *stands for the greatest integer function, then*

(a) $f\left(\frac{\pi}{2}\right) = -1$ (b) $f(\pi) = 1$ (c) $f(-\pi) = -1$ (d) $f\left(\frac{\pi}{4}\right) = 2$

$\pi^2 = (3.14)^2 = 9.7$

$[\pi^2] = 9.$

$[-\pi^2] = -10.$



$f(x) = \cos 9x + \cos[(-10)x] = \cos 9x + \cos 10x.$

$f\left(\frac{\pi}{2}\right) = \cos 9\frac{\pi}{2} + \cos 5\pi = -1$

$f(\pi) = \cos 9\pi + \cos 10\pi = -1 + 1 = 0$

$f(-\pi) = 0$

$f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + 0 = \frac{1}{\sqrt{2}}$

ILLUSTRATION 2 *Let* $f(x) = [x]^2 + [x + 2] - 8$, *where* $[x]$ *denotes the greatest integer less than or equal to* x, *then*

(a) $f(x) \neq 0$ *for all* $x \in R$ ✗

(b) $f(x) = 0$ *only for two real values of* x

(c) $f(x) = 0$ *for infinitely many values of* x ✓

(d) *none of these*

$[x+n]$ where n = natural no

$[1.2+2] = 3 = [1.2]+2$

$[1+2] = 3 = [1]+2.$

$[x+n] = [x]+n.$

$f(x) = [x]^2 + [x] - 6.$

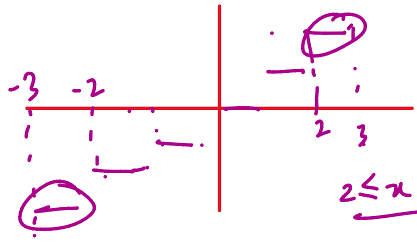
$a^2 + a - 6 = (a+3)(a-2)$

$0 = \{[x]+3\}\{[x]-2\}$

$[x] = -3, 2.$

$\downarrow \quad \downarrow$

$-3 \le x < 2$



$2 \le x < 3$

infinitely many real values of x.

ILLUSTRATION 3 *If for a real number* x, $[x]$ *denotes the integral part of* x. *Then, the value of*

$$\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{100}\right] + \left[\frac{1}{2} + \frac{2}{100}\right] + \ldots + \left[\frac{1}{2} + \frac{99}{100}\right], \text{ is } = \left[100 \times \frac{1}{2}\right] = 50$$

(a) 49 (b) 50 ✓ (c) 48 (d) 51

$[0.5] + [0.5 + \frac{1}{5}] + [0.5 + \frac{2}{5}] + [0.5 + \frac{3}{5}] + [0.5 + \frac{4}{5}]$

$0 + 0 + 0 + 1 + 1 = \underline{\underline{2}} = \boxed{2.5} = \left[5 \times \frac{1}{2}\right]$

$[x + \frac{0}{5}] + [x + \frac{1}{5}] + [x + \frac{2}{5}] + \ldots + [x + \frac{4}{5}]$

$[1 + \frac{0}{5}] + [1 + \frac{1}{5}] + [1 + \frac{2}{5}] + \ldots + [1 + \frac{4}{5}]$

$x = 1$ $\quad 1 + 1 + 1 + \cdot 1 \cdot + 1$

$= 5 = [5 \times 1] = [5 \times x]$

$x > 1$ $\quad [1.5 + \frac{0}{5}] + [1.5 + \frac{1}{5}] + [1.5 + \frac{2}{5}] + [1.5 + \frac{3}{5}] + [1.5 + \frac{4}{5}]$

$1 + 1 + 1 + 2 + 2.$

$= 7 = [7.5] = [5 \times 1.5]$

Any $[x + \frac{0}{n}] + [x + \frac{1}{n}] + [x + \frac{2}{n}] + \cdots + [x + \frac{n-1}{n}] = [nx]$

ILLUSTRATION 4 *If for a real number x, [x] denotes the greatest integer less than or equal to x, then for any* $n \in N$

$$\left[\frac{n+1}{2}\right] + \left[\frac{n+2}{4}\right] + \left[\frac{n+4}{8}\right] + \left[\frac{n+8}{16}\right] + \ldots =$$

(a) n (b) $n-1$ (c) $n+1$ (d) $n+2$

$n = 1$

$1 + 0 + 0 \ldots = 1$

$n = 17$

$9 + [\frac{19}{4}] + [\frac{21}{8}] + [\frac{25}{16}] + [\frac{33}{32}] + [\frac{49}{64}] + \cdots$

$9 + 4 + 2 + 1 + 1 + 0. = 17$

ILLUSTRATION 6 *The number of integral solutions of the equation* $\{x+1\} + 2x = 4[x+1] - 6$, *is*

(a) 0 (b) 1 (c) 2 (d) 3

$\{x\} \rightarrow$ fraction part of x.

$[x+1] + 2x = 4[x+1] - 6$, is

(a) 0 (b) 1 (c) 2 (d) 3

$\{x\} \to$ fraction part of x.

$[x] \to$ gint(x)

$x = 1.3$.

$x = 1 + 0.3$.

$\boxed{x = [x] + \{x\}}$

$\{x\} = x - [x]$

$$\{x+1\} + 2x = 4[x+1] - 6$$

$$x+1 - [x+1] + 2x = 4[x+1] - 6$$

$$5[x+1] = 3x + 7$$

$$[x+n] = [x] + n.$$

$$5([x]+1) = 3x + 7$$

$$5[x] = 3x + 2.$$

for $\underline{x = \text{integer}}$ $[x] = x$.

$$5[x] = 3[x] + 2.$$

$$2[x] = 2.$$

$$[x] = 1$$

$[x] = 1$

$1 \leq x < 2$

ILLUSTRATION 2 *The domain of the functions:*

$$f(x) = \frac{1}{\sqrt{|x| - x}}, \text{ is}$$

(a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $R - \{0\}$ (d) *none of these*

$$|x| - x > 0.$$

$$\boxed{|x| > x.} \to x = ?$$

$$x < 0$$

$[x] - |x| > 0.$ $[x] > |x|$

$|x| - [x] > 0.$ $|x| > [x].$

$[x] - x > 0$

$[x] > x.$

$\underline{x = 1.5}$

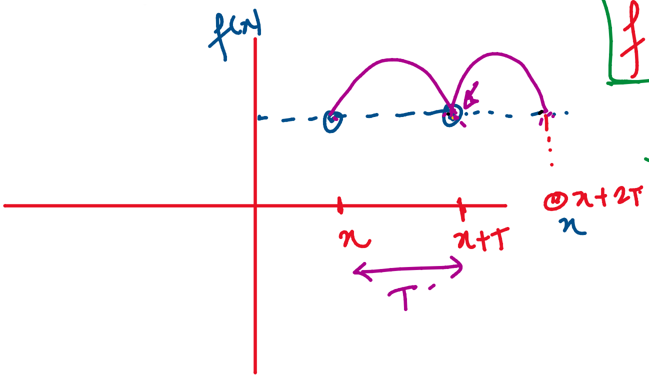
$[x] = 1$

$x = -1.5$ $[x] = -2$

ILLUSTRATION 1 *The function* $f(x) = x - [x]$ *is a periodic with period.*

(a) 1 (b) 2 (c) 3 (d) *none of these*

$y = |\sin x|$



$\boxed{f(x+T) = f(x)}$

Period

$\boxed{f(x) \text{ is continuous}}$

$f(x)$ will have a maxima/ minima between x and $(x+T)$

$f(x+2T) = f(x+T) = f(x)$