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Number Theory

How to find ① Last Two digits

How to calculate the remainder if a large exponent?

$21^{68} \rightarrow$ _ _

$$(20+1)^{68} = \left[20^{68} + {}^{68}C_1\, 20^{67} + {}^{68}C_2\, 20^{66} + \cdots + {}^{68}C_{66}\, 20^{2}\right] + \left\{ {}^{68}C_{67}\, 20 + 1 \right\}$$

00 00 00 00 . + 1

$68 \times 20 + 1$

$= 1360 + 1 = 1361$

Rule: $\boxed{\text{If the last digit of the base} = 1}$ then multiply the 2nd last digit of the base and the last digit of the power. The last digit of this no will be the 2nd last digit of the expansion

21^{68} $2 \times 8 = 16$

391^{256} 41 $9 \times 6 = 59$

$43^{68} \rightarrow (\_\_\_1)^{?}$ $3^{6} \rightarrow (\_\_1)^{?}$

$3^{98} \rightarrow 3^{4 \times 24 + 2}$ $3^4 = 81$

$= (3^4)^{24} \times 3^2$ $3^6 = 81 \times 3^2$

$= (81)^{24} \times 9$ $= 81 \times 9$

$= 21 \times 9 = 89$ 29

$43^{68} = (40+3)^{68} \rightarrow {}^{68}C_{67}\, 40\, 3^{67} + 3^{68}$

17

$3^{67} = 3^{4\times16+3} = (3^4)^{16} \times 3^3$

$= (81)^{16} \times 27$

$= 81 \times 27 = 2187$

$= 68 \times 40 \times 87 + (81)^{17}$

$= 40 + 61$

$= 01$

$(37)^{34} \rightarrow 34(30)7^{33} + 7^{34}$

$= 1020 \times 7^{33} + 7^{34}$

$\boxed{7^4 = 2401}$

$= 1020 \times 7^{4\times8+1} + 7^{4\times8+2}$

$= 1020\,[(01)^8 \times 7] + (01)^8 \times 49$

$= 1020\,[7] + 49 = 140 + 49$

$= 89$

$(29)^{36}$

$\rightarrow \boxed{36 \times 20 \times 9^{35} + 9^{36}}$

$= 720 \times 49 + 41$

$= 80 + 41$

$= 21$

$\boxed{9^2 = 81} \leftarrow \quad 35 = 2\times17+1$

$9^{35} = 9^{2\times17} \times 9$

$= (81)^{17} \times 9$

$= 61 \times 9$

$= 49$

$(81)^{18}$

$= 41$

Last digit - 1, 3, 7, 9

$2^{56} \rightarrow \_\_\_$

$\boxed{2^{10} = 1024}$

$= 2^{10\times5} \times 2^6$

Rule any number ending in 24 odd

$(24) \rightarrow 24$

even $\quad 76$

$= 2 \times 2$

$= (24)^5 \times 64$

$= 24 \times 64 = 36$

$(24) \to 24$

$(24)^{even} \to 76$

(*) any no ending in 76 will always end in 76.

$2^1 = 2$

$2^2 = 4$

$2^3 = 8 = 10 - 2$

$2^4 = 6 = 10 - 4$

$2^4 = 16$ | $2^8 = 56$ | $2^{12} = 96$ | $2^{16} = 36$ | 76
$2^5 = 32$ | $2^9 = 12$ | $2^{13} = 92$ | $2^{17} = 72$ | 52
$2^6 = 64$ | $2^{10} = 24$ | $2^{14} = 84$ | $2^{18} = 44$ | 04
$2^7 = 28$ | $2^{11} = 48$ | $2^{15} = 68$ | $2^{19} = 88$ | 08

$(42)^{26} = 2^{26} \times 21^{26}$

64×21

44

$2^{10 \times 2} \times 2^6$

$76 \times 64 \to 64$

$4^{90} \to$

$4^5 = 1024$

$8^{90} \to 2^{270}$

5, 6

$6^{90} \to 2^{90} \times 3^{90}$

$15^2 = 25$

$15^3 = 75$

$15^4 = 25$

$15^5 = 75$

$(15)^{even} \to 25$

$(15)^{odd} \to 75 = 100 - 25$

$35^{even} \to 25$

$35^{odd} \to 75$

$25^2 = 25$

$25^3 = 25$

$35^2 = 25$

$35^3 = 75$

$35^4 = 25$

$35^5 = 75$

$25^2 = 25$ $\quad$ $35^2 = 25$ $\quad$ $55 = 25$

$25^3 = 25$ $\quad$ $35^3 = 75$ $\quad$ $35^5 = 75$

Rule 3

$(15, 35, 55, 75, 95)^{even} \Rightarrow 25$

$\boxed{(15, 35, 55, 75, 95)^{odd} \rightarrow 75}$

$(25, 45, 65, 85, 05)^{any} \rightarrow 25$

$\left(\underline{x} \; 1\right)^{abc} \rightarrow \underline{\underline{\text{last of } x \times c}} \; 1$

$(-2)^n \rightarrow$ last 2 terms $\rightarrow$ use $2^{10} = 1024$

$(24)^{odd} = 24$

$(24)^{even} = 76$