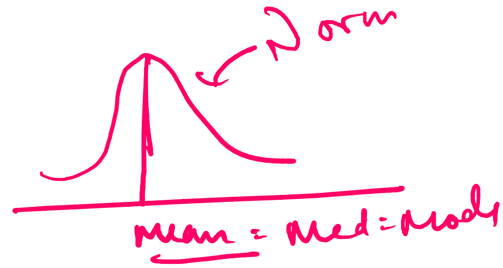
13 October 2023 07:33 PM

condition for pdf.

1. $f(x) \geq 0$ for all x.
2. $\int_{-\infty}^{\infty} \underbrace{f(x)}\, dx = 1$

↳ For a normal distribution.



Properties of Normal distribution:

1. The distribution is symmetrical.

and Mean = Median = Mode = μ.

$E(x)$

$f(x)$

2. $Var(x) = \sigma^2$

3. $\mu_3 = 0$, $\mu_4 = 3\sigma^4$

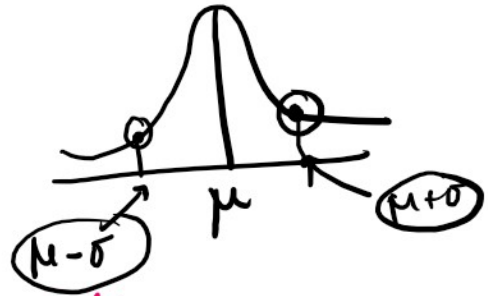
(all odd order central moments of N.D is zero)

4. $\beta_1 = 0$ and $\beta_2 = 3$

Skewness $\gamma_1 = 0$ Kurtosis $\gamma_2 = 0$.

5. Point of inflection of the normal ... bell-shaped

(5) point of inflexion of curve, which is bell-shaped are $x = \mu \pm \sigma$



(6) about 99.73% of variate value lie in the interval $(\mu - 3\sigma, \mu + 3\sigma)$ which is called effective range of distribution

(1) f(x) in case of ND is a pdf.

Proof $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$; $-\infty \, x \, \infty$

Here (i) $f(x) > 0$ for all value of x.

(ii) $\int_{-\infty}^{\infty} f(x)\,dx$

$= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$ dx

$= \int_{-\infty}^{\infty} \frac{1}{\cancel{\sigma}\sqrt{2\pi}} e^{-\frac{1}{2}t^2} \cancel{\sigma}\cdot dt$

$\boxed{t = \frac{x-\mu}{\sigma}}$

$dt = \frac{dx}{\sigma}$

$\boxed{dx = \sigma \cdot dt}$

$= \int^{\infty} e^{-\frac{1}{2}t^2} dt$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}t} dt$$

$\hookrightarrow$ even fn

integration of even fn

$\because e^{-t^2/2}$ is " "

$$\int_{-\infty}^{\infty} f(x)\,dx = \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-\frac{t^2}{2}} dt$$

let $\frac{t^2}{2} = y =$

$t^2 = 2y$

$t = \sqrt{2y}$

then $\cancel{2}t\,dt / \cancel{2} = dy$

$dt = \frac{dy}{t}$

$dt = \frac{dy}{\sqrt{2y}}$

$$= \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-y} \frac{dy}{\sqrt{2y}}$$

$$= \frac{2}{\sqrt{2\pi}\sqrt{2}} \int_{0}^{\infty} e^{-y} y^{-1/2} dy$$

$$\int_{0}^{\infty} e^{-x} x^{-\alpha} dx = \Gamma\alpha$$ ← gamma distribution

$$\boxed{\Gamma 1/2 = \sqrt{\pi}}$$

$$= \frac{2}{\sqrt{2\pi}\sqrt{2}} \Gamma 1/2$$

$$= \frac{2}{\sqrt{2\pi}\sqrt{2}} \times \sqrt{\pi}$$

$$= \frac{\cancel{2}}{\cancel{2}\cancel{\sqrt{\pi}}} \times \cancel{\sqrt{\pi}}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Proved

$$\int_{-\infty} f(x)\,dx = 1$$

② Mean of Normal Distribution:

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let $t = \frac{x-\mu}{\sigma} \rightarrow x = \mu + \sigma t$

$\boxed{\sigma\, dt = dx}$

$$E(x) = \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sigma t)\, e^{-t^2/2} \cdot dt\, \cancel{\sigma}$$

$$= \underbrace{\frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2/2}\, dt}_{} + \underbrace{\frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} t \cdot e^{-t^2/2}\, dt}_{0}$$

odd fn

[: integration of odd fn = 0]

$$= \frac{2\mu}{\sqrt{2\pi}} \int_0^{\infty} e^{-t^2/2}\, dt + 0$$

Let $y = t^2/2$

$dy = \frac{2t}{2}\, dt$

$dt = \frac{dy}{t}$

$dt = \frac{dy}{\sqrt{2y}}$

$$= \frac{2\mu}{\sqrt{2\pi}} \int_0^{\infty} e^{-y}\, y^{-1/2} \frac{dy}{\sqrt{2}}$$

$$= \frac{2\mu}{\sqrt{2\pi} \cdot \sqrt{2}} \times \Gamma\tfrac{1}{2}$$

$$= \frac{2\mu}{} \times \sqrt{\pi}$$

$$= \frac{\cancel{2}\mu \;\; \sqrt{2\pi}\cdot\sqrt{2}}{\cancel{2}\sqrt{\pi}} \times \cancel{\sqrt{\pi}}$$

$\sqrt{2}\,4$

$\therefore E(x) = \mu$ (ans)

③ even order central moment: $\mu^{2r} = E(x-\mu)^{2r}$

$$\mu^{2r} = \int_{-\infty}^{\infty} (x-\mu)^{2r} f(x)\,dx$$

$$\mu^{2r} = \int_{-\infty}^{\infty} (x-\mu)^{2r} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$