14 July 2023 07:59 PM

Q1. The nine digits $1, 2, 3, \ldots, 9$ are arranged in random order in the form of a nine-digit number. Find the probability that 1, 2 and 3 appear as neighbours in the order mentioned

7 ways 6 places 6!

(1,2,3) - - - - - 1
- (1 2 3) - - - - 2
- - (1 2 3) - - -

7 ways 6!
9!

required probability

$$= \frac{7 \times 6!}{9!} = \frac{\cancel{7}}{9 \times 8 \times \cancel{7}} = \frac{1}{72} \text{ (ans)}$$

Q2: A club consisting of 15 married couples chooses a president and a secretary by random selection. What is the probability

(i) both are men (ii) one man and other is women 2!

(iii) the president is a man & the secretary is a woman.

1 15 1 15

$^{30}P_2 = 30 \times 29$ (two positions occupied by 30 persons).

(i) 15 men occupying 2 positions i.e. $^{15}P_2 = 15 \times 14$

$\therefore$ required probability is $\frac{\cancel{15} \times \cancel{14}\,7}{\cancel{2}\cancel{30} \times 29} = \frac{7}{29}$

(ii) favourable event $^{15}C_1 \times {}^{15}C_1 \times 2!$

(ii) favourable event $^{15}C_1 \times {}^{15}C_1 \times 2!$
$= 15 \times 15 \times 2$

$\therefore$ Required probab is $\dfrac{15 \times \cancel{15} \times \cancel{2}}{\cancel{2}\,\cancel{30} \times 29} = \dfrac{15}{29}$

(iii) required probability $= \dfrac{\cancel{15} \times 15}{\cancel{30} \times 29} = \dfrac{15}{58}$

Q3 A box contains 7 white and 5 black balls. Three balls are drawn at random find probability that they are all of same colour when (i) the balls are drawn at a time (ii) one by one without replacement (iii) one by one with replacement.

(i) $^{12}C_3 = 220 \rightarrow$ total number of elementary events.

$^7C_3 + {}^5C_3 = 35 + 10 = 45 \rightarrow$ favourable to event that all are of same colour since all may be white or all may be black.

$\therefore$ Required probability
$1 - P$ (all are same colour)
$= 1 - 45/220 = 1 - \dfrac{9}{44} = \dfrac{35}{44}$

(ii) Total elementary events $= {}^{12}P_3 = 12 \times 11 \times 10 = 1320$

favourable events $\rightarrow {}^7P_3 + {}^5P_3 = 270$

Required probability is $1 - \frac{270}{1320}$

(iii)

Required probability: $1 - \frac{7^3 + 5^3}{(12)^3}$

$\frac{7}{12}\frac{7}{12}\frac{7}{12}$

$+ \frac{5}{12}\frac{5}{12}\frac{5}{12}$

Q5 Sum of points will be even (A_1) or ($\cup$) less than 5 (A_2).

$6^2 = 36$ (Total events)

A_2: $(1,1)$ $(1,2)$ $(1,3)$ $(2,1)$ $(2,2)$ $(3,1)$

$P(A_1) = \frac{18}{36}$ $P(A_2) = \frac{6}{36}$ $P(A_1 \cap A_2) = \frac{4}{36}$

Req prob: $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$= \frac{18}{36} + \frac{6}{36} - \frac{4}{36}$

$= 20/36 = 5/9$

Independent event:
$P(A \cap B) \neq P(A)P(B)$

Q6 $P(A) \rightarrow$ spade $\Rightarrow 13/52$

$P(B) \rightarrow$ King $\Rightarrow 4/52$

$P(A \cap B) \rightarrow$ King & spade $\Rightarrow 1/52$

$P(A) \times P(B) = \frac{\cancel{13}}{\cancel{52}} \times \frac{\cancel{4}}{\cancel{52}} = \frac{1}{52}$

$\therefore P(A \cap B) = P(A)\,P(B)$

$\therefore$ A and B are indep events.

Q7 In a group of 20 males and 5 females, 10 males and 5 females are service holders. What is the probability that a person selected at random from the group is a service holder (A), given that the selected person is a male (B)

A → service holder

B → male

$P(B) = 20/25 = 4/5$

$P(A \cap B) = 10/25 = \frac{2}{5}$

$P(A/B) = ? \quad \frac{P(A \cap B)}{P(B)} = \frac{2/5}{4/5}$

$= 1/2$ (ans)

Q8 Three boxes → black and white balls

A_1 Box I : 5 black & 3 white

A_2 Box II : 6 black & 2 white

A_3 Box III : 3 black & 5 white

(i) Prob of selecting P(B) black ball

(ii) Given ball is black find prob that it is selecting from box II

$P(A/B)$

$P(A_1) = P(A_2) = P(A_3) = 1/3$

$P(B/A_1) = 5/8 \quad P(B/A_2) = 6/8 \quad P(B/A_3) = 3/8$

(i) $P(B) = P(A_1) \cdot P(B/A_1) + P(A_2)P(B/A_2) + P(A_3)P(B/A_3)$

$= \frac{1}{3} \times \frac{5}{8} + \frac{1}{3} \times \frac{6}{8} + \frac{1}{3} \times \frac{3}{8}$

$= 14/24 = 7/12$ (ans).

(ii) $P(A_2/B) = \dfrac{P(A_2)\,P(B/A_2)}{P(B)}$ (Bay's theorem)

$$P(A_2/B) = \frac{P(A_2 \cap B)}{\sum_{i=1}^{3} P(A_i)\,P(B/A_i)} = \frac{P(A_2).P(B/A_2)}{P(B)}$$

$$= \frac{1/3 \times 3/8}{7/12} = \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{8}_2} \times \frac{\cancel{12}^{3}}{7} = 3/14$$