Domain and Range

$f(x) = {}^{3x-1}C_{20-4x}$

$3x-1 \geq 20-4x$

$\Rightarrow x \geq 3$

$3x-1 \geq 0 \quad \& \quad 20-4x \geq 0$

$x \geq 1/3 \qquad x \leq 5$

$x \in [1,\infty), \quad x \in [1,5], \quad x \in [3,\infty)$

$\Rightarrow \quad x \in [3,5] \qquad x \in \{3,4,5\}$

HW $\quad f(x) = \log_x 5 + \sqrt{\cos(\sin x)}$ $\quad$ domain – $(0,1) \cup (1,\infty)$

$R^+ - \{0,1\}$

Ranges

$f(x) = a\cos x + b\sin x$

Range $= \left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$

$f(x) = \sin^{2n} x + \cos^{2n} x$

Range $= \left[\frac{1}{2^{n-1}}, 1\right]$

$f(x) = \sin^{2n+1} x + \cos^{2n+1} x$

Range $= [-1, 1]$

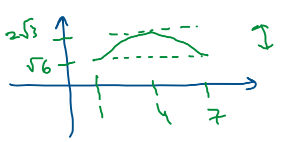
$f(x) = \sqrt{x-1} + \sqrt{7-x}$

Domain $\rightarrow$ $[1,7]$

$f'(x) = \frac{1}{2}\left[\frac{1}{\sqrt{x-1}} - \frac{1}{\sqrt{7-x}}\right]$

Set $f'(x) = 0$, we have $x = 4$

$f(1) = f(7) = \sqrt{6}, \quad f(4) = 2\sqrt{3}$ $\quad$ Range $= [\sqrt{6}, 2\sqrt{3}]$



Operation of odd & even functions

$f(x)$	$g(x)$	$f(x)+g(x)$	$f(x)-g(x)$	$f(x)\cdot g(x)$	$f(x)/g(x)$	$(fog)x$
Even	Even	Even	Even	Even	Even	Even
Even	Odd	Neither Even nor Odd	Neither Even nor Odd	Odd	Odd	Even
Odd	Even	Neither Even nor Odd	Neither Even nor Odd	Odd	Odd	Even
Odd	Odd	Odd	Odd	Even	Even	Odd

Periodic Functions

Properties → $f(x) \to$ periodic. Period $\to p$.

(1) $af(x)+b \to$ periodic with period p

$a \neq 0$ (2) $f(ax+b) \to$ periodic with period $p/|a|$

(3) $g(x) \to$ period q, $f(x)+g(x) \to \text{LCM}(p,q)$

(4) $1/f(x) \to$ period p

(5) $\sqrt{f(x)} \to$ period p

(6) $g(f(x)) \to$ period p, if g is monotonic

(7) $g \circ f(x)$ is periodic, $f \circ g(x)$?

no guarantees

$$f(x) = \frac{e^{x}-e^{-x}}{2}$$. Find $f^{-1}(x)$

→ let $x_1, x_2 \in \mathbb{R}$, $f(x_1) < f(x_2)$ for $x_1 < x_2$

$x_1 < x_2 \Rightarrow e^{x_1} < e^{x_2}$ as $e > 1$

$-x_2 < -x_1 \Rightarrow e^{-x_2} < e^{-x_1}$ as $e > 1$

Adding → $e^{x_1} - e^{-x_1} < e^{x_2} - e^{-x_2}$

$\Rightarrow f(x_1) < f(x_2)$